CORRECTED INPUTS—A METHOD FOR IMPROVED HYBRID SIMULATION

Robert Gelman
General Electric Company
Re-Entry Systems Department
Philadelphia 1, Pennsylvania

I. INTRODUCTION

The purpose of this paper is to describe a programming procedure which is designed to minimize some of the difficulties often encountered in hybrid simulations. The method has been worked out for the hybrid system at General Electric's Re-Entry Systems Department, in Philadelphia. This system consists of general purpose analog and digital machines, connected by analog to digital (A-D) and digital to analog (D-A) converters, and associated logical elements. The procedure itself will be referred to as the method of corrected inputs. It is designed to minimize three major sources of error connected with data transfer. These are:

1. *D-A Data*

   Due to the nature of the digital computer, each variable which it sends to the analog is seen by the analog as a stepped function. This introduces errors because of the differences between this function and the actual one, and also because of the response of the analog to discontinuous inputs. Filtering this input can smooth the discontinuities somewhat, but introduces lags and distortions.

2. *A-D Data*

   The analog output is sampled and sent to the digital computer no more than once during each digital computation interval. If the digital time step is larger than, or even a significant portion of, the fundamental frequency of the analog output, then the A-D data could actually be a very poor representation of the analog outputs.

3. *Time Lags*

   The minimum time required for an output from a subsystem to affect the operation of the subsystem itself would be the sum of the A-D and D-A time lags. This could render hybrid operation very difficult, if not completely useless, in studies of the performance and stability of control systems, or other studies involving leads, lags, or phase relationships.

The method of corrected inputs provides a simple way of minimizing all these difficulties. The basic idea behind the method is to represent, on the analog, some of the functions which are also calculated on the digital computer. The outputs of the digital simulation would then be used, not as inputs to the main analog simulation, but as corrections to the comparable analog portion.

II. DESCRIPTION OF METHOD

The following is a brief summary of the system. The timing is arranged so that the digital computations lag the analog by the digital computation interval. Those inputs to the main analog program which are sensitive to phase, delays, frequency response, or similar factors, are obtained from subsections of the analog program. These inputs are sampled and sent to the digital computer, where they are stored until the digital simulation produces the corresponding quantities, presumably with more accuracy. The differences are then returned to the analog computer which generates corrections, in the form of ramp functions, to the analog inputs. The only requirement imposed on the analog subsystems involved is that the analog functions not diverge appreciably from the comparable digital functions in less than two digital computation intervals.

The analog inputs to the digital program are not sent directly, unless they are slowly varying functions. Otherwise, each one is averaged over the time period of one computation interval, and this average is sent to the digital. The fact that the digital calculations lag the analog enables the digital computations to be made using inputs from the analog which are the average values for the period of time covered by the calculations. Thus, the method minimizes the three major drawbacks to accuracy in hybrid simulations: data transfer time lags, discontinuous D–A signals, and unrepresentative A-D signals.

The manner in which this method can be used to meet the basic difficulties described above can best be understood if it is explained in more detail. This is done with the aid of figures 1, 2, and 3.

Figure 1 is a schematic representation of a hybrid simulation. The box F_1 represents, let us say, the low frequency portion of the prob-

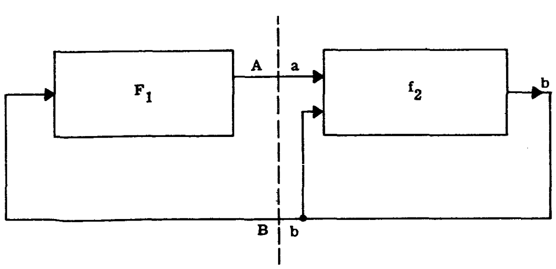

Figure 1. Schematic representation of a hybrid simulation.

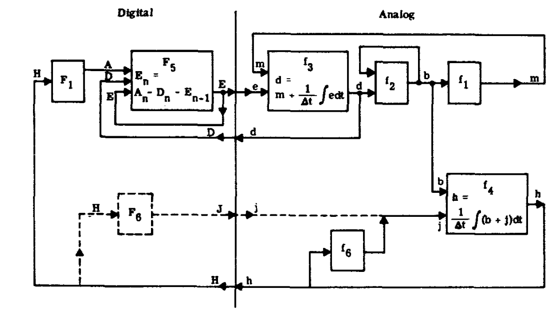

Figure 2. Hybrid simulation, modified for method of corrected inputs.

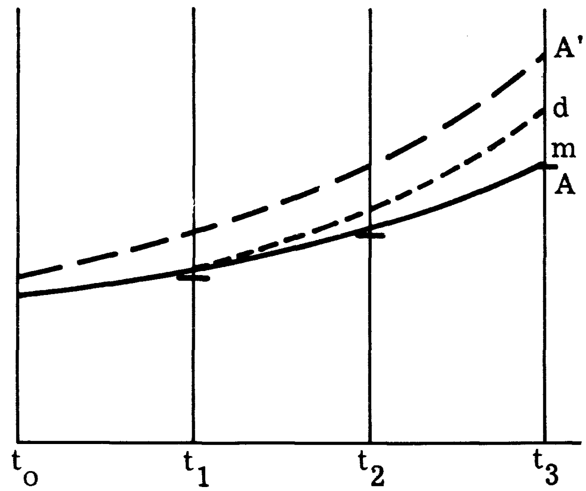

Figure 3-A. Variables during corrected D-A data transfer.

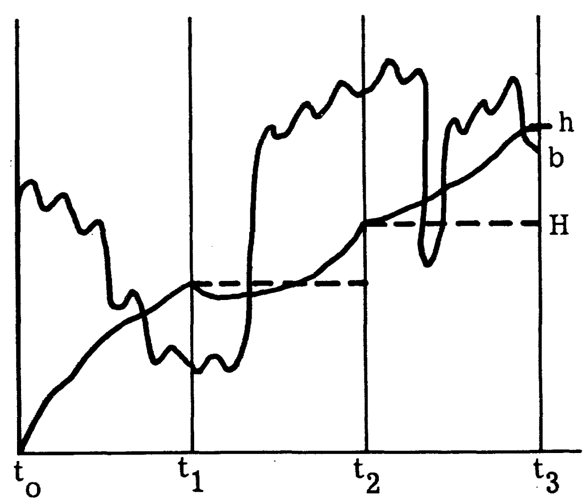

Figure 3-B. Variables during A-D data averaging procedure.

lem, solved on the digital computer, and box f_2 represents the high frequency portion, simulated on the analog. Output b is shown feeding back into f_2 in order to illustrate the possibility that the behavior of f_2 can depend on high frequency as well as low frequency inputs. Note that all digital functions are represented by capital F's, analog functions by small f's, digital variables by capital letters, and analog variables by small letters. When the same letter is both capitalized and lower case, it represents either a variable that is converted A-D or D-A, or a function which is simulated on both computers.

Figure 2 shows the simulation of figure 1, as modified for the method of corrected inputs. The modified circuit is not as much more complicated than the original one as these diagrams would make it appear. The analog function, f_3, consists of two amplifiers; f_4 consists of one. The digital operation F_5 consists of summing three numbers; F_6 consists of moving one number from an input location to an output. The analog function f_1 could be complex, but experience shows that it is usually possible to generate, by very simple means, a function which will be close enough to F_1 for our purposes. The function f_6, if used, is a sample and hold circuit.

The D-A data transfer will be explained first. The normal hybrid simulation would convert A directly to its analog equivalent, a, and feed it into f_2, as shown in figure 1. In the method of corrected inputs, f_2 receives its basic input, the quantity m, from f_1, as shown in figure 2. A correction is added to m before it is fed into f_2, so that f_2 receives the quantity d, which is a corrected m. The function f_3 is an integration and an addition. That is,

$$d = m + \int e\,dt \tag{1}$$

The behavior of the variables is shown in figure 3-A. The dashed line, A′, shows what A would look like if it were calculated continuously and in phase with the analog. At time t_0, the quantity d is sampled as d_0, which is equal to m_0. It is converted to the digital number D_0, and stored in digital memory. At time t_1, the quantity A_0 has been computed, and the difference, E_0 $(=A_0 - D_0)$, is converted to e_0, and fed into f_3. Thus, for the period from t_1 to t_2,

$$d = m + (A_0 - D_0)\frac{t - t_1}{t_2 - t_1} \tag{2}$$

Also at t_1, the quantity d_1 $(=m_1)$ is converted to D_1 and stored. Then, at t_2, the quantity $A_1 - D_1$ is computed. Here we have to consider one of the more subtle points of the process. The quantity D_1 does not reflect the correction, e_0, which has already been added to m.* For illustrative purposes, consider the special case, $m = A' - K$ is a constant. Then we would find that

$$A_0 - D_0 = K \tag{3}$$

$$A_1 - D_1 = K \tag{4}$$

$$A_2 - D_2 = 0 \tag{5}$$

If we use, for E, the general formula

$$E_n = A_n - D_n, \tag{6}$$

then we would obtain for consecutive values of d,

$$d_0 = A' - K \tag{7}$$

$$d_1 = A' - K \tag{8}$$

$$d_2 = A' \tag{9}$$

$$d_3 = A' + K \tag{10}$$

$$d_4 = A' + K \tag{11}$$

$$d_5 = A' \tag{12}$$

$$d_6 = A' - K \tag{13}$$

etc.

If we use the formula

$$E_n = A_n - D_n - E_{n-1}, \tag{14}$$

we would obtain for consecutive values of d,

$$d_0 = A' - K \tag{15}$$

$$d_1 = A' - K \tag{16}$$

$$d_2 = A' \tag{17}$$

$$d_3 = A' \tag{18}$$

etc.

Thus, the subtraction of the preceding correction term removes a phase lag oscillation from the D-A input. It is apparent from the foregoing that a condition is imposed on f_1, requiring that the amount that m diverges from A

* From the procedure, it follows that $d_n = m_n + \sum_{i=0}^{n-2} e_i$

in the time $t_{n+2} - t_n$, is no greater than the acceptable error in d. That is, from the time the variable d is sampled to the time the corresponding correction has been added to it, two time cycles have elapsed. Thus, as long as the inaccuracies in the generation of m do not cause it to drift more than an acceptable amount in two time cycles, it will be satisfactory.

The A-D data transfer is illustrated in figure 3-B. The variable, b, is shown as having high frequency components, and large excursions, so that samples taken at each digital time interval might not be representative. The solution consists of using b to construct another function, h, whose value at any sampling time, t_n, is equal to the average value of b during the interval $t_n - t_{n-1}$. The resulting value, h_n, is converted to H_n, and fed into F_1. The digital computer, at time t_n, is just starting the computations for the problem interval between t_{n-1} and t_n. All the variables will have values corresponding to time t_{n-1}, except those coming from the analog, which will have values equal to what their average will be during the time interval, $_{n-1}$ to t_n, for which the computations are to be made. It is to be noted that any high speed system responses are automatically reflected in b because of the closed analog loop through f_1, f_3, and f_2.

The manner in which h is produced from b is, in principle, very simple. In the first time interval, from t_0 to t_1, the only input to f_4 is b. Assuming that $t_n - t_{n-1}$ is a constant,† then it is a simple matter to adjust f_4 such that,

$$h_1 = \int_{t_0}^{t_1} b\,dt/(t_1 - t_0). \tag{19}$$

The quantity h_1 is sampled, and fed back as soon as possible as j_1. Thus, at time t_2, we find,

$$h_2 = h_1 + \int_{t_1}^{t_2} b\,dt/(t_2 - t_1) - \int_{t_1}^{t_2} j_1\,dt/(t_2 - t_1). \tag{20}$$

But, the first and third terms on the right side of this expression cancel out, leading to the general result,

† This assumption is not necessary, but the use of varying time increments would require the use of nonlinear analog equipment; specifically, a multiplier.

$$h_n = \int_{t_{n-1}}^{t_n} b\,dt/(t_n - t_{n-1}). \tag{21}$$

The manner in which h is fed back to f_4 as j will affect the timing of the data transfer, as well as the accuracy of h. The output, h, can be sampled and fed directly back to f_4, by adding a sample and hold circuit, shown as f_6, to the analog simulation. Or, h can be sent to the digital computer as H, sent back immediately as J, and converted to j. I prefer the former method, as it leads to a neater data transfer routine. Therefore, the timing sequence which follows assumes that f_6 is in the circuit, and F_6 is out. The alternate sequence, based on F_6 in the circuit and f_6 out, is included as appendix A.

Figure 4-A illustrates the timing sequence involved in the operation of the method of corrected inputs, assuming the inclusion of f_6 in the analog simulation. The actual D-A data transfer will occur in a time interval τ seconds

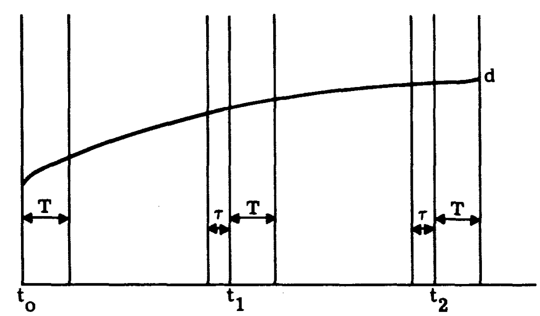

Figure 4-A. Timing sequence if analog averaging circuit is used.

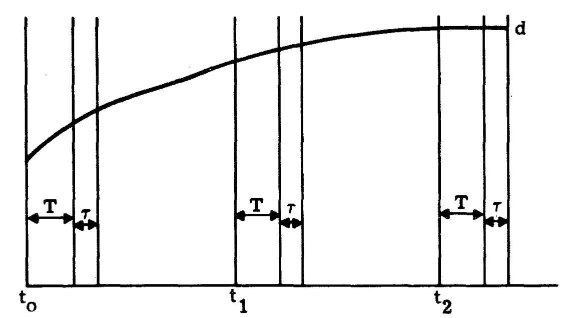

Figure 4-B. Timing sequence if hybrid averaging circuit is used.

long, and the A-D transfer in an immediately subsequent interval lasting T seconds. The sequence of operations follows:

$\underline{t_0}$

Analog variables sampled. (d_0 and h_0)

h_0 (=0) fed back to f_4

$\underline{t_0 \rightarrow (t_0 + T)}$ (A-D data transfer)

d_0 ($= m_0$) $\rightarrow D_0$

h_0 (=) $\rightarrow H_0$

$\underline{(t_0 + T) \rightarrow (t_1 - \tau)}$ (Digital computations)

$E_0 = A_0 - D_0$

$\underline{(t_1 - \tau \rightarrow t_1}$ (D-A data transfer)

$E_0 \rightarrow e_0$

$\underline{t_1}$

Analog variables sampled. (d_1 and h_1)

$$h_1 \left(= \int_{t_0}^{t_1} bdt/(t-t) \right) \text{ fed back to } f_4$$

$\underline{t_1 \rightarrow (t_1 + T)}$ (A-D data transfer)

d_1 ($=m_1$) $\rightarrow D_1$

$h_1 \rightarrow H_1$

$\underline{(t_1 + T) \rightarrow (t_2 - \tau)}$ (Digital computations)

$E_1 = A_1 - D_1 - E_0$

$\underline{(t_2 - \tau) \rightarrow t_2}$ (D-A data transfer)

$E_1 \rightarrow e_1$

$\underline{t_2}$

Analog variables sampled. (d_2 and h_2)

$$h_2 \left(= \int_{t_1}^{t_2} bdt/(t_2 - t_1) \right) \text{ fed back to } f_4$$

$\underline{t_2 \rightarrow (t_2 + T)}$ (A-D data transfer)

d_2 ($=m_2 + e_0$) $\rightarrow D_2$

$h_2 \rightarrow H_2$

III. RESULTS OF AN ANALOG DEMONSTRATION

A simplified three degree of freedom simulation of the flight and control of a missile was chosen to illustrate the method of corrected inputs. A description of the simulation is given in appendix B. This work was done on the analog, because the digital computer which comprises half of the hybrid facility is being replaced, a process which will not be completed until early in 1964. In order to simulate a hybrid operation on an analog computer, it was necessary to make simplifications, compromises, and omissions. Even so, it is felt that a valid and informative demonstration was obtained. The main differences between this analog demonstration and a full scale hybrid simulation are:

1. The data transfer took place at much lower rates. Most of the runs used transfer rates between one and ten cycles per second, instead of the 0.1 to 1 KC rates that are customarily used. This was not due to limitations of the equipment, but to the fact that it is much easier to demonstrate the method at the lower rates.
2. The digital equations were simulated on the same time base as the analog equations, with no lag. It is easy enough to store a value in the digital computer until the time comes to use it, but it was not felt to be worth the effort to accomplish the same thing on the analog.
3. The A-D averaging procedure was set up and run, but it was not used in the full simulation, as the fact that the digital lag was not included would have made it a hindrance, rather than a help. Figures 5 and 6 show examples of its operation at 1 and 10 cps respectively.

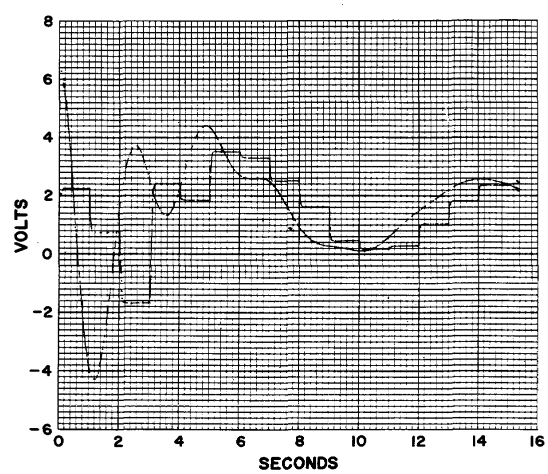

Figure 5. L, averaged and sampled at 1 cps.

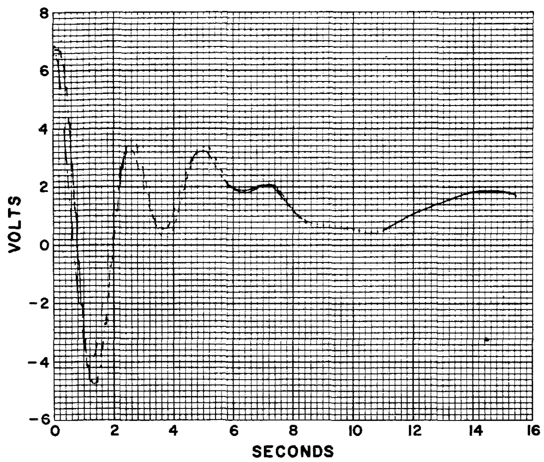

Figure 6. L, averaged and sampled at 10 cps.

Figure 7 is a schematic of the analog demonstration. The nominal case, against which the others can be compared, is set up when both switches are in position 1. Both switches are put in position 2 to represent a standard hybrid simulation. Putting switch A in position 4 makes the circuit represent an all analog simulation, and putting it in position 3 includes the D-A correction circuit.

Figure 8 shows the recorder outputs corresponding to switch B in position 1, and switch A in positions 1, 2, and 3, respectively. Sampling rate was 2 cps.

Figure 9 shows the variable, y, measured at points 1, 4, 3, and 2 of switch A (figure 7) with both switches in position 1. The sampling rate was 2 cps. Comparison of the third and fourth curves of this figure illustrates the difference between a sampled and a corrected input. Note that the approximate function differs appreciably from the controlling one.

Figures 5 and 6 show the operation of the A-D correction circuit. Figure 5, obtained with a one cps sampling rate, illustrates how each sampled value of the output is equal to the average value of the input function during the preceding sample and hold cycle.

In order to get some numerical indication of the effectiveness of each procedure, a function, Q, was generated, such that

$$Q = k \int y^2 \, dt$$

where the effectiveness is considered to be inversely proportional to Q. Table 1 shows the results of measuring Q at the termination of each run. All the runs stopped automatically, at x = 9000 ft. A comparison of the first three columns of this table is shown in figure 10. The data in these curves were taken without A-D sampling, so that they are a measure of the type of D-A transfer used, and of the sampling frequency. Curve A shows the results of D-A transfer, and represents an optimum result, obviously independent of sampling frequency. Curve B plots results with corrected D-A inputs, which can be seen to be far superior to the results obtained by sampling the D-A inputs, shown as curve C.

IV. SUMMARY

In summary, let us see how the method of corrected inputs minimizes the three sources of error listed at the beginning of this paper.

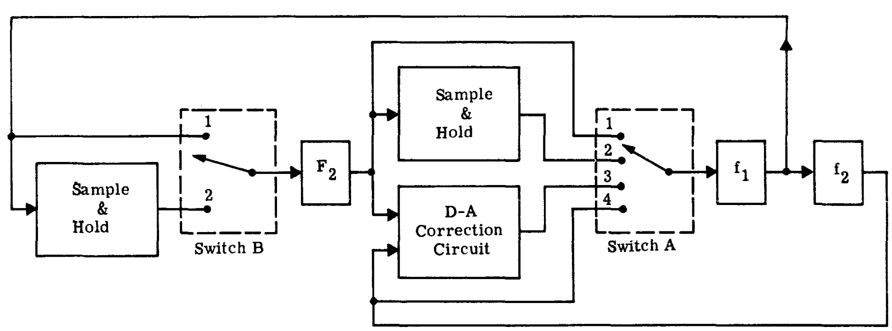

Figure 7. Schematic diagram of analog simulation of method of corrected inputs.

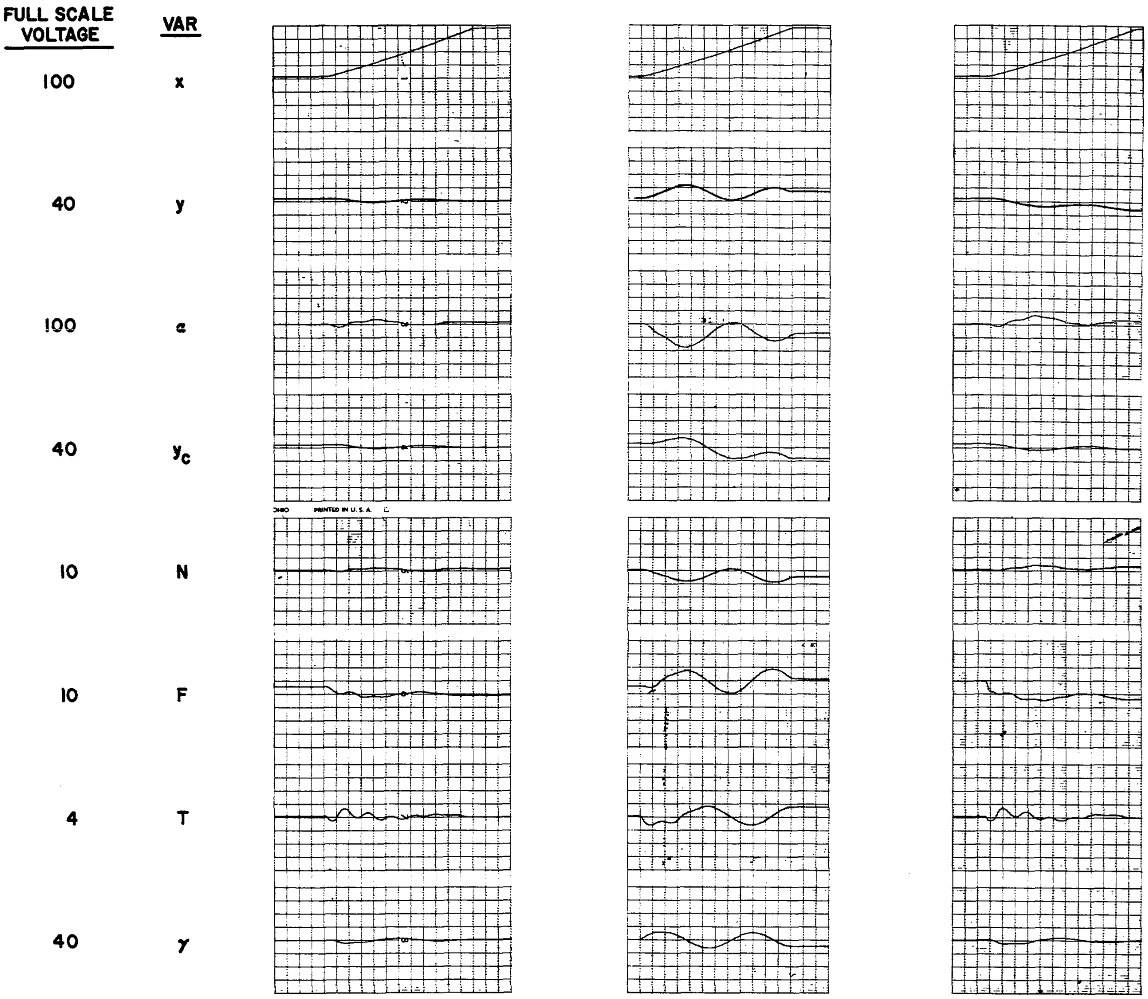

Figure 8. Recorder outputs for runs with direct D-A, sampled D-A, and corrected D-A, respectively (two cps).

1. *D-A Data Transfer Errors*
   The input to the main analog program is smooth instead of stepped. It is in phase with the simulation into which it is fed. It is constantly corrected; the correction is no more than two digital computation intervals behind the actual problem.

2. *A-D Data Transfer Errors*
   The digital computer receives the analog variable, already averaged over the interval for which the digital computations are to be made, instead of sampled at its value at the start of that interval. Any errors introduced by the averaging process will tend to cancel themselves out, rather than accumulate.

3. *Time Lags*
   The simulation can respond immediately to variations in any portion of the system. This ability is completely independent of either the A-D or the D-A transfer time requirements.

Thus, we have a system which ameliorates difficulties which were thought by many to be intrinsic to hybrid simulation.

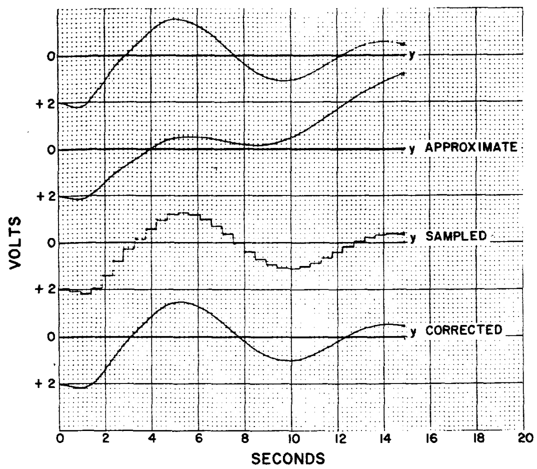

Figure 9. Variable, y, measured at various points during nominal run.

APPENDIX A

ALTERNATE TIMING SEQUENCE FOR METHOD OF CORRECTED INPUTS

The timing sequence shown in this appendix would be used in the event that the correction term j, which is fed to f_4 (see figure 2), originates in the digital computer, rather than the analog circuitry. The comments refer to figure 4-B. All A-D data transfers occur in the time intervals T, and all D-A data transfers occur in the immediately subsequent time intervals τ.

$\underline{t_0}$

Analog variables sampled. (d_0 and h_0)

$\underline{t_0 \rightarrow (t_0 + T)}$ (A-D data transfer)

$d_0 \ (= m_0) \rightarrow D_0$

$h_0 \ (=_0) \rightarrow H_0$

$\underline{(t_0 + T) \rightarrow (t_0 + T + \tau)}$ (D-A data transfer)

$J_0 \ (= H_0) \rightarrow j_0$

$\underline{(t_0 + T + \tau) \rightarrow t_1}$ (Digital computations)

$E_0 = A_0 - D_0$

$\underline{t_1}$

Analog variables sampled. (d_1 and h_1)

$\underline{t_1 \rightarrow (t_1 + T)}$ (A-D data transfer)

$d_1 \ (= m_1) \rightarrow D_1$

$h_1 \left(= \int_{t_2}^{t_1} bdt/(t_1 - t_0) \right) \rightarrow H_1$

$\underline{(t_1 + T) \rightarrow (t_1 + T + \tau)}$ (D-A data transfer)

$J_1 \ (= H_1) \rightarrow j_1$

$E_0 \rightarrow e_0$

$\underline{(t_1 + T + \tau)} \rightarrow$ (Digital computations)

$E_1 = A_1 - D_1 - E_0$

$\underline{t_2}$

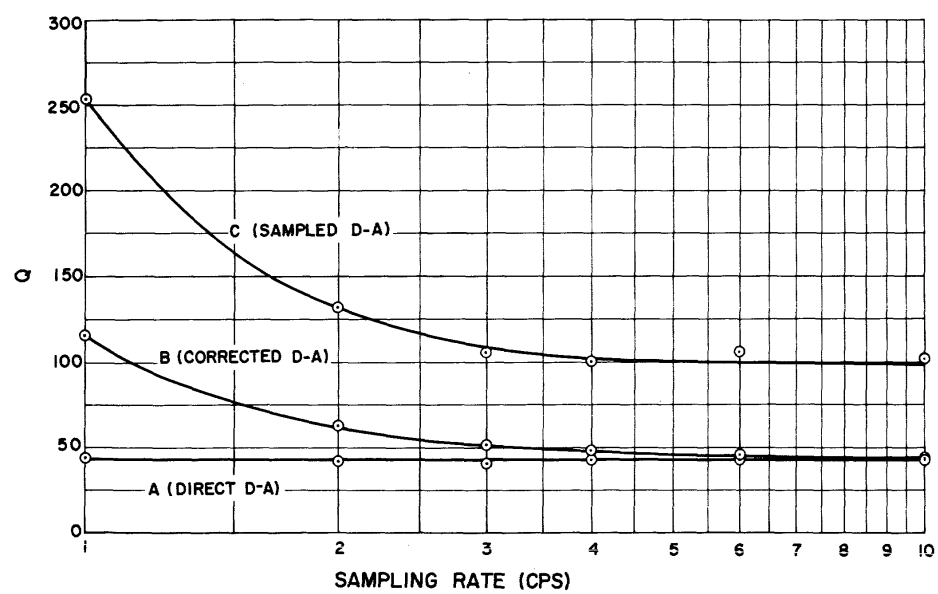

Figure 10. Q as a function of D-A sampling method and sampling rate.

Analog variables sampled. (d_2 and h_2)

$\underline{t_2 \rightarrow (t_2+T)}$ (A-D data transfer)

$d_2 \ (=m_2+e_0) \rightarrow D_2$

$$h_2 \left(= \int_{t_1}^{t_2} b\,dt/(t_2-t_1) \right) \rightarrow H_2$$

$\underline{(t_2+T) \rightarrow (t_2+T+\tau)}$ (D-A data transfer)

$J_2 \ (=H_2) \rightarrow j_2$

$E_1 \rightarrow e_1$

APPENDIX B

ANALOG SIMULATION TO DEMONSTRATE METHOD OF CORRECTED INPUTS

This all analog simulation was split into so-called "digital" and "analog" portions, to represent those parts of the problem which would normally be assigned to each computer. The equations for calculating the acceleration, velocity, and position vectors were considered digital, and the equations involving the control system, applied forces and torques, and rotational dynamics were considered analog.

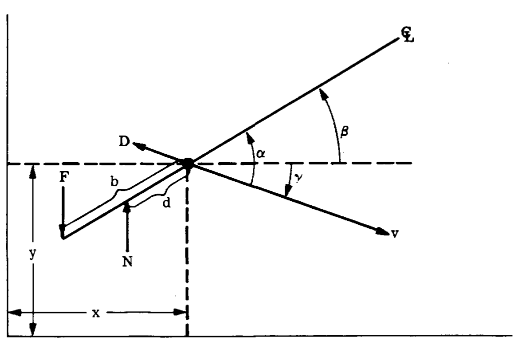

Figure 11. Diagram showing symbols used for dimensions and variables in the sample problem.

Referring to figure 11, the analog equations are:

$$\ddot{\beta} = -\,bF/I - dN/I - T/I \qquad \text{(B-1)}$$

$$\dot{\beta} = \dot{\beta}_0 + \int \ddot{\beta}\,dt \qquad \text{(B-2)}$$

$$\beta = \beta_0 + \int \dot{\beta}\,dt \qquad \text{(B-3)}$$

$$a = \beta - \gamma^{\ddagger} \qquad \text{(B-4)}$$

$$N = k_1\,\alpha\,\dot{x}^2 \qquad \text{(B-5)}$$

$$F = k_3 y^{\ddagger} + k_4 \dot{y}^{\ddagger} + k_5 \beta + k_6 \dot{\beta} \qquad \text{(B-6)}$$

$$T = k_2 \dot{\beta} \dot{x}^{\ddagger} \qquad \text{(B-7)}$$

$$L = N + F \qquad \text{(B-8)}$$

and the digital equations are:

$$\ddot{x} = -\,D/M + k_7 \qquad \text{(B-9)}$$

$$\dot{x} = \dot{x}_0 + \int \ddot{x}\,dt \qquad \text{(B-10)}$$

$$x = x_0 + \int \dot{x}\,dt \qquad \text{(B-11)}$$

$$D = k_8 x^2 \qquad \text{(B-12)}$$

$$y = y_0 + \int \dot{y}\,dt \qquad \text{(B-13)}$$

$$y = \dot{x}\gamma \qquad \text{(B-14)}$$

$$\dot{\gamma} = L^{**}/m\dot{x} \qquad \text{(B-15)}$$

$$\gamma = \gamma_0 + \int \dot{\gamma}\,dt \qquad \text{(B-16)}$$

The quantities y, $\dot{y}$, and γ are approximated by a section of the simulation which replaces equations B-13, B-14, B-15, and B-16 with the equations:

$$y = y_0 + \int \dot{y}\,dt \qquad \text{(B-17)}$$

$$\dot{y} = \dot{y}_0 + \frac{1}{m}\int L\,dt \qquad \text{(B-18)}$$

$$\gamma = \dot{y}/\dot{x} \qquad \text{(B-19)}$$

For this simulation, the values of the constants were:

$m = 1.555 \times 10$	pound-sec²/ft
$I = 6.22 \times 10$	pound-ft-sec²
$b = 5$	feet
$d = 2$	feet
$k_1 = 3 \times 10^{-4}$	pound-sec²/ft²
$k_2 = 1.73 \times 10^{-3}$	pound-sec²
$k_3 = 1.06 \times 10$	pounds/ft
$k_4 = 5.8$	pound-sec/ft
$k_5 = 7.16 \times 10$	pounds

‡ The quantities y, $\dot{y}$, γ, and $\dot{x}$ are received from the digital portion of the simulation.

§ The quantity L is received from the analog portion of the simulation.

$k_6 = 1.83 \times 10$ pound-sec

$k_7 = 3 \times 10$ feet/sec^2

$k_8 = 1.25 \times 10^{-4}$ pound-sec^2/ft^2

In order to simulate the operation of the corrected D-A inputs, the variables y, $\mathring{y}$, and γ were sent through the circuit of figure 12-A (the D-A correction circuit box of figure 7). In this figure,

- v = the desired function
- v' = the approximate function
- v_s = the output of the sample and hold circuit
- v_f = the correcting and feedback voltage
- v_c = the final corrected output

A1, A2, and A4 are summing amplifiers, and A3 is an integrator. In operation, the potentiometer, P, is set so that

$$\mathring{v}_f = v_s/T_c,$$

where T_c is the period of the sample and hold circuit. The result is that v_f is a ramp voltage whose value at the end of each hold period is equal to the difference between v and v′ at the time of sampling. The problem was run using this correction circuit, both with and without sampling the variable L.

The A-D correction circuit (not the sampling circuit) is shown in figure 12-B. The output, v_c, is equal, during the hold period, to the average value of the input, v, during the preceding sample and hold period. This circuit, while operative (see figures 5 and 6), was not used in conjunction with the entire simulation.

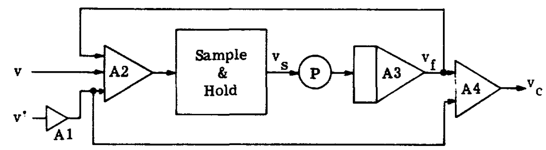

Figure 12-A. Analog simulation of D-A correction circuit.

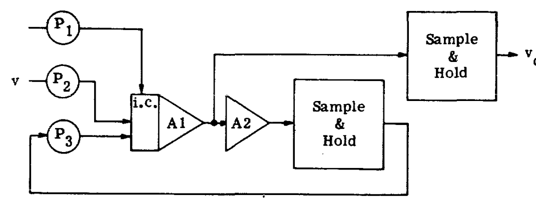

Figure 12-B. Analog simulation of A-D correction circuit.

TABLE 1—VALUES OF Q

Type of data transfer		Sampling Rate, cps					
D-A	A-D	1	2	3	4	6	10
Direct	Direct	44	42	41	43	44	42
Sampled	Direct	254	132	105	100	106	102
Corrected	Direct	116	63	51	48	46	41
Approx.	Direct	810	726	736	800	707	892
Direct	Sampled	272	126	100	96	94	94
Sampled	Sampled	—*	—*	266	207	171	164
Corrected	Sampled	890	334	196	161	135	123
Approx.	Sampled	—*	—*	—*	—*	—*	—*

* Unstable, went into overload.