EXPERIENCE WITH HYBRID COMPUTATION

E. M. King, Jr., and R. Gelman
Missile and Space Division
General Electric Company
Philadelphia 1, Pennsylvania

INTRODUCTION

In recent years, there has been increasing emphasis on the use of electronic analog and digital computers in the fields of science, engineering, and education. Interest has also been generated in computational methods not readily available from either of these classes of computers alone, but rather from the combination of them provided by the so called hybrid computer system. This interest has been a direct result of the increase in the complexity of present day systems and hardware, and the increased accuracy required of them. Based on two years of experience with a large hybrid facility, this paper will review the main characteristics of digital and analog computers, compare their capabilities and their limitations, and show some of the reasons for and advantages of mating the two to form the hybrid computing system. We will describe some of the problems to which we have successfully supplied hybrid solutions, and predict the direction of future hybrid progress.

Simulation

In general, there are two distinct modes of simulation; mathematical and physical.

Mathematical simulation utilizes a mathematical model of the physical system under study. This model is excited by certain ordered inputs and provides information about the behavior of the physical system, to the extent that it is correctly represented by the mathematical model.

Physical simulation requires the excitation of the system under conditions which are representative of those encountered in actual system operation. This testing can involve anything from an inclined plane to large multi-million dollar ventures like the Space Environmental Simulator located at General Electric's Valley Forge, Penna., Space Technology Center. These two types of simulation can be combined by mating physical hardware with a mathematical model.

The general purpose computers available today are primarily designed for mathematical simulation. They are ordinarily quite capable of simulating our physical systems with a depth and accuracy comparable to that obtainable by building and testing the actual equipment. As such, the general purpose analog and digital computers have allowed engineering to advance further and faster than ever before. It is, therefore, a definite requirement that any new entrant to these fields become familiar with these basic machines; their abilities, their limitations, and their possible use in his endeavors.

The Analog Computer

An electronic analog computer is an array of computational building blocks, or modules, each being able to perform a particular mathematical operation on an input voltage signal and provide a specific output response. These building blocks normally provide the functions of summation, integration with respect to time, multiplication by a constant, multiplication and division of variables, function generation, generation of trigonometric functions, and representation of system discontinuities. All quantities are represented on the analog by continuously varying voltages, restricted on almost all analog computers to the range between -100 and +100 volts.

The ease with which this computer is able to perform the operation of integration makes it ideally suited to the solution of differential equations, as the differential equations can be integrated a suitable number of times to form integral equations, and these integral equations can be represented on the computer. Data are fed into the analog computer in the form of parameter settings, which are usually associated with the coefficients that exist in the mathematical equations. Data are extracted from the computer in the form of voltages, either as steady-state values which can be read out on a voltmeter, or as varying values which can be recorded on a strip chart recorder or a plotting table. Some of the analog characteristics pertinent to our discussion are:

1. The analog is a parallel machine. All the variables are computed simultaneously and continuously. Thus, the speed with which the calculations are made is completely independent of the size or complexity of the problem.
2. The bigger a problem is, the more equipment is needed, as each piece of equipment works on one part of the problem.
3. Numbers on the analog are fixed point. Every variable must be scaled. The scaling will greatly affect the accuracy of the results.
4. The analog is best suited for solving systems of ordinary linear differential equations, although it can handle many other types of problem in a very satisfactory way.
5. There is no such thing as a computational cycle with the analog, because of characteristic No. 1. The analog can be set to calculate at any rate desired, but in practice there is an optimum time base associated with any particular problem, and attempts to run the problem much faster <u>or slower</u> will severely degrade the accuracy. The analog, generally speaking, is faster than the digital.
6. Analog outputs are almost always accurate to within 1%, but seldom better than 0.1%.
7. It is very easy, with most problems, to introduce extensive changes in the simulation in a matter of minutes.

Principal Areas of Use

Although the analog computer was designed primarily for the solution of problems in the aircraft field, its area of application has broadened considerably over the years. It has become a major analysis and synthesis tool in the electrical, mechanical, and chemical industries, and as a tool of instruction in our leading universities interested in graduating well-rounded engineers.

While a list of applications of the analog computer would provide a sizeable volume itself, suffice it to say that the analog computer can be applied profitably to almost any problem that can be specified in the form of mathematical equations. The exact form may require a special programming technique, but information is readily available on most of these. In addition, these special cases are welcomed by the applications engineer as a continuation of his education.

The Digital Computer

The digital computer works by a counting technique and obeys logic rules exactly. The solutions are at discrete points dependent on the size of the time increment used. The smaller the mesh size, the more we approach the continuous solution. In contrast to the analog computer, which uses continuous variables in the form of voltages, the digital computer uses discrete variables, and operates with numbers as opposed to voltages. The digital computer is essentially a very fast calculating machine. It is able to perform simple arithmetical operations of addition, subtraction, multiplication, and division. By re-writing the mathematical equations representing a physical system or a physical problem in a special form, it is possible to program a digital computer to solve the same kind of problems that are solved on the analog computer by simulation. The power of the digital computer lies in the fact that theoretically it is capable of solving any problem that can be written in the form of simple arithmetical operations.

There are a number of digital computer characteristics that are of particular interest in connection with hybrid simulation. These are:

1. It will deal only with numbers. Any problem must be reduced to a series of numerical operations before it can be handled

by the computer. This is not to say that every step must actually be written each time. All sorts of aids to compiling programs are available. A program is nothing more than the entire sequence of instructions given to the computer to solve a problem. In actual practice, the machine itself will write most of its own instructions.

2. It will do exactly what it is told. All changes involve writing new instructions. The easier it is to make a change, the more complicated the original instructions have to be to include the option.

3. The results are exactly repeatable, but their accuracy is dependent on the numerical methods used to solve the problem.

4. The computer will perform only one operation at a time. That is, if the instruction reads, "Move number N from location A to location B," the machine will, for a given period of time, be doing nothing but that.

5. The computer works with increments. None of the variables are calculated continuously. Generally speaking, the larger the calculation increment of the digital computer, the faster and the less accurate is the computation. There is absolutely no drift with a digital computer.

6. Compared with an analog, the digital is very much better equipped to make decisions. These can be made on the basis of comparison, time, reaching a point in the program, or almost any other criterion chosen by the programmer.

7. The digital can store very much more information than the analog. It can store tables, functions of several variables, whole programs, and many other things.

Principal Areas of Use

It is almost impossible to list the areas of application of the computer because of the diversity involved. We can say, however, that the digital computer lays sole claim to those problems which store a lot of information, use much logic, or require extreme accuracy. It will calculate trajectories, solve problems in astronomy, simulate mental processes such as learning and memory, analyze games, do translations, help design new computers, and do untold numbers of other tasks. The major effort to discover new computer applications is devoted to the digital area, with the analog a poor second, and the hybrid far behind.

Table 1 is a comparison of the relative performance of analog and digital computers with respect to some operating characteristics which might interest a user. The

Table 1

Characteristic	Analog			Digital		
	Ex	Good	Poor	Ex	Good	Poor
Accuracy (See discussion)						
Speed	X				X	
Absence of long term drift			X	X		
Adaptability to change in program	X					X
Information storage			X	X		
Decision making			X	X		
Ease of programming	X				X	
Ability to solve differential equations	X			X		
Ability to simulate control functions	X					X
Ability to tie in with hardware	X					X
Ability to operate in real time	X				X	
Repeatability		X		X		
Amount of output data available		X		X		
On line outputs	X				X	
Accuracy of outputs			X	X		
Changing outputs	X				X	
Changing inputs	X				X	
Systematic parametric variation		X		X		

information included is intended only as a rough guide to general tendencies, and a stimulus to discussion. It can't possibly be more than that, for several reasons. First, there is a very great variation in characteristics from computer to computer. This is particularly true of digital computers, where the speed, for instance, can vary by a factor of a million, and great disparities exist between the principles of operation. There are significant variations in analog computer characteristics as well. The table was compiled on the basis of experience with large, up-to-date installations, both digital and analog, and undoubtedly reflects this fact. Second, there are no generally accepted standards by which the listed characteristics can be evaluated. Obviously, one per cent accuracy will be excellent for some problems, while one part in a billion will be poor for others. This particular list is highly subjective, and it's very unlikely that anybody else would put his X's in all the same boxes we have. One can always think of a special case which will move any X into any box. Even so, we feel fairly confident that most people who have worked with both types of computer would be in sufficient agreement with this chart to justify its use as an aid to someone considering hybrid simulation for the first time.

The subject of accuracy is so complicated, and dependent on so many factors, that it just didn't seem possible to summarize it by a mark in a box. While this is to some extent true of all the other characteristics listed, we believe considerations of accuracy fall into a special case.

On an analog computer, the result is usually within 0.1% and 1% of the value inherent in the equations. Whether this is excellent or poor depends on the nature of the problem. In many engineering investigations, this is much more precise than the data upon which the problem is based. The use to which the answer will be put also affects the accuracy required. Determination of the region of stability of a control system to within a millionth of the control range would be valueless, as the nature of the input could affect it much more than that. On a digital computer, the ultimate limit of accuracy is the number of bits in a word. This accuracy is seldom attained by the output variables of a problem, due to the approximations involved in almost any mathematical model, the idiosyncrasies of programming, and the practical necessity of taking reasonably large computing steps. The question concerning accuracy is more often, "How much cost and effort is needed to obtain the required accuracy?", than "What accuracy is obtainable?" The answer has to be determined separately for each individual problem.

Combined Analog-Digital Computer Systems

Having duly noted most of the characteristics of analog and digital computers independently, we come to a recurrent question in the industry today, "Is there a need for a new computer system, and if so, what should it be?"

Actually the first part seems to be unanimously answered in the affirmative, while the second leads to all sorts of answers, such as:

1. More digitized analog computers
2. More flexible analog type modules for digital computers
3. Digital differential analyzers
4. Hybrid system coupling the present day analog and digital computers.

This last seems to have the largest support and will receive ours as well. As justification for this support, we can cite our own experience with the hybrid facility at GE MSD; the installation itself, some of the reasons for developing it, and specific examples of its profitable use.

This experience has been with a hybrid installation which combines two large, general purpose computers; one digital, and one analog. It is used primarily for simulation of missile and space vehicle systems and components. Other hybrid facilities are considerably different. Each one is a custom installation, designed for particular needs. Some, as ours, connect a general purpose digital computer to a general purpose analog. Some use a digital differential analyzer rather than an analog. At least one installation uses all three types of computer. The hybrid used at the GE Defense Systems Department in Syracuse, New York, is semi-specialized, as it is used for many types of problems, but only in the field of electronic system simulations. Despite the breadth of the field, there are enough uses and characteristics common to most hybrid installations to make much of what is said here generally applicable, although limited.

There are three obvious reasons for the development of a new type of computation. These are:

1. It will provide a more satisfactory means of performing some calculations than is presently available.

2. It will do computations that would not be practical at all with existing methods.

3. It appears to have potentialities that are worthwhile exploring, if only to keep abreast of competition.

This last reason has, in some cases, undoubtedly been a large factor in the decision to develop a hybrid system, competitive pressures being what they are. However, considerations of this kind do not seem to be pertinent to this particular presentation, and so will not be considered further.

The advantages of a hybrid that we felt to be of most value to the work of the department were in the area of increasing the size and variety of the problems we could solve. The things a hybrid can do to help in that endeavor are:

1. Assign different sections of a problem to each computer. For instance, in simulating a missile, the trajectory calculations can be assigned to the digital, because of the available precision, and the control simulation put on the analog because of its flexibility.

2. Assign different functions to each computer. For instance, all integrations might be assigned to the analog computer, in order to save time and get a continuous output. Or, all function generation might be assigned to the digital computer (where it is known as table look-up).

3. Provide analog plots of digital variables. This is particularly useful in observing the behavior of selected variables while the simulation is in progress. In one case, a stop was put on a 7090 after the first 15 seconds of what would otherwise have been a 10 minute run because it was easy to tell from the behavior of a continuous analog output that a key variable was not behaving quite as desired.

4. Let the digital provide logic for the analog. Things such as switching, scale changing, ending the program, choosing tables to examine, can be readily programmed into the digital and can greatly simplify and possibly even speed up an analog simulation.

5. Allow real hardware to be part of a simulation. Most hardware can readily be connected into the analog, and hybrid operation would allow it to connect to the digital just as easily. Similarly, digital devices can be included in analog operation the same way. Real hardware could also be considered to include people, as part of a control loop.

6. Provide accurate digital printouts of analog variables. Normally, the accuracy with which the analog variables are plotted is less than the accuracy that actually exists in the equipment. Hybrid operation enables selected variables to be converted to digital form and printed out from a digital tape. This gives an accurate printout of an analog variable in any of the forms available for digital variables.

There are many other things that can be accomplished by hybrid computation, but perhaps it would be best at this point to give some examples of problems which were actually done this way.

Examples of Hybrid Simulations

The examples given here are all taken from work done at the hybrid facility at General Electric's Missile and Space Division in Philadelphia. Each illustrates a particular type of use to which such a facility can be put. The explanations will necessarily be brief, but it is hoped they will be sufficient for our illustrative purposes.

Voice Data Reduction: This task consisted of reconstituting, from a digital tape, the original signals from which the tape had been made. The hybrid problem was the last link in a larger study, investigating the conditions under which a spoken message could be converted into digital signals and still contain enough information to be reconverted into intelligible speech. The main study was conducted by another company (Sylvania) which supplied the digital tape to us. The input tape was used as an input to a digital program which scaled the signals and sent them to Hycol (HYbrid COmputing Link) in the proper time sequence. The output of the Hycol was recorded on tape, which when played back produced the original message intelligibly. The program was a straightforward one involving only a one way transfer of information and not making use of the analog computer at all. Actually, part of the program did send the input data back to the digital computer for checking purposes, but the primary information flow was in one direction; D-A. The program was run in real time. Had it been necessary to transmit information at a greater rate, the data comparison portion of the program might have had to be curtailed or omitted.

Telemetry Data Reduction: This task was very similar to the voice tape program described, in that data were supplied to the program by a digital tape which was used as the primary input of a digital program which, in turn, sent data to the Hycol in real time. In this case, the purpose of the program was to provide a tape which could be used to check the operation of telemetry data reduction equipment. The original tape contained coded signals corresponding to known time variations of all the quantities to be measured. The hybrid program received this data, placed it in the core memory of the digital computer, and performed the operations necessary to dole it out to Hycol in real time, properly coded. The Hycol output was sent through a filter which put the signal in the form required by the telemetry data reduction equipment. This procedure had two advantages over other methods of testing the equipment. One, the input signal was just what it would be expected to receive in actual operation. It even included errors at chosen points. Two, the output could be compared directly with the inputs to the original computer program.

Missile and Telemetry Simulation: The object of this hybrid simulation was, as with the preceding one, to test a telemetry data reduction system. The output of the program consisted of twenty voltages, equal to those that would be produced by the transducers on a missile under the conditions being simulated. In order to produce this output, two separate computer programs were required. The first was a six degree of freedom all-digital simulation which generated the parameters that would be measured by on-board instrumentation in a real flight. The complexity of the program made it necessary for it to generate the data at less than the real time rate. A second program used these data as inputs, scaled them, and converted them to analog signals which were used as real time inputs to an analog simulation. The analog provided the inputs to the telemetry equipment by simulating the behavior of the transducers aboard the vehicle. As in most hybrid simulations at this facility, all the D-A signals were not only sent to the analog, but were reconverted and sent back to the digital for checking purposes. In this simulation, the final output of the data reduction system could be compared directly with the output of the all-digital missile dynamics program.

Missile Control and Trajectory Simulation: This simulation was part of a preliminary design effort whose object was to develop a very accurate short range tactical missile. The part of the design effort involving hybrid simulation was the study of the effect of parametric variations and control system design on the impact accuracy of the missile. This was investigated by a full hybrid simulation, involving the simultaneous operation of both computers. The missile control system was simulated on the analog, and the trajectory, dynamics, and aerodynamics on the digital. The machines did their calculations at the same time, with each one sending and receiving information during an assigned portion of each digital computing cycle. The usual check of the data transfer was made by sending the D-A data back, via A-D, to the digital, and printing out both sets of numbers. This use of the hybrid enables us to have the flexibility and convenience of the analog for investigating the control system, and at the same time use the data storage capacity and precision of the digital for the dynamics and the trajectory calculations. This particular simulation could not be run in real time because of the complexity of the six degree of freedom digital program.

SUMMARY

Some hybrid computer applications with which we have had personal experience have been described. Other hybrid installations have reported work on chemical processes, flight analysis and control, air traffic control, reaction jet control, solution of partial differential equations, and function generation, among others.

The advantages of hybrid simulation are based on the disparate nature of analog and digital computers. The characteristics of the analog that are most useful in hybrid operation are:

1. The variables are in the form of easily available d.c. voltages, continuously varying in time.
2. The program can be changed very rapidly and easily.

3. The output is recorded as the problem is run.
4. Transfer functions and feedbacks can be represented very easily.
5. It is very easy for most engineers to get the physical "feel" of a problem on the analog.
6. The analog can usually operate very fast, and the speed is independent of the size of the problem.

The aspects of the digital which are very useful in hybrid simulation are:

1. Precision
2. The ability to store information
3. The ability to make logical decisions
4. Stability and repeatability
5. Printing out large amounts of data in tabular form.

It is obvious that any problem which would benefit by the availability of computer characteristics from both lists could probably benefit from being set up as hybrid problem.

As far as the future of hybrid operation is concerned, we feel that the biggest contribution will be made by the digital computer manufacturers, although this has not been the case so far. Eventually, some digital computers will come with optional packages which will enable at least these three instructions to be added:

1. Stop until time t on clock, or priority interrupt at time t, then proceed.
2. Transfer to analog, from core location X, to output channel Y, scale value Z.
3. Transfer from analog, to core location X', from input channel Y', scale value Z'.

The package will contain an accurate source of time signals, n output jacks, and m input jacks. The jacks could be connected directly to the analog computer.

The first instruction will either stop the computer until a specified time signal is received, or send it to a chosen instruction at that time.

The second instruction will take the (presumably) floating point number in core location X, convert it to fixed point, and convert it to a voltage which will be established at output jack Y. The value of the voltage will depend on the scale value. For instance, if a quantity 2.3×10^5 is stored in core location X, and the scale value Z (that is, the value represented by full scale voltage, which is 100 volts on most analogs) is 5×10^5 per 100 volts, then 46 volts would appear on output jack Y. As many as n transfer out instructions could be given during one computer cycle.

The third instruction would transfer analog data into the digital computer in a manner similar to the way the second instruction transfers out. As many as m transfer in instructions would be possible per computing cycle.

With these three simple instructions, a great variety of hybrid simulations could be readily performed. It would not be necessary to limit the information transferred in this manner to problem variables, as a voltage appearing on a trunk line of an analog can be used for any purpose desired, such as starting, changing modes of operation, and operating relays. It would be expected, however, that the transfer of problem variables would utilize more of the transfer package than any other function, for most problems. The simplicity of the programming procedures would probably be a greater factor than any other one thing in encouraging people to avail themselves of the advantages of both types of computer in a single problem, instead of limiting themselves to either type alone.

We feel that a listing of installations provided as part of the first lecture at a recent hybrid symposium at Electronic Associates Computation Center in Princeton, New Jersey is well worth including as an indication of the variety of systems in use, and as a source of further information.

Table 2

Location	Digital Computer	Linkage
IBM - Yorktown Heights	IBM 704	EAI
University of Minnesota	Univac 1103	EAI
Ramo Wooldridge originally IBM for Ft. Belvoir - Fieldata	PB 250	EAI
Douglas Aircraft	G - 15	EAI
White Sands Missile Range	IBM 704 (7090 later)	EAI
General Motors Goleta, California	CDC 160 A	EAI
Minneapolis Honeywell Pottstown, Pennsylvania	MH 290	EAI
Detroit Tank Arsenal	Datatron 205	EPSCO A-D EAI D to A only
Western Electric	Univac thru Bell Tel. I/O	EAI D to A
Westinghouse Electric	Special West. Computer	EAI D to A Plotters
Convair Astronautic Vacuum Tube	IBM 704	EPSCO
Space Technology Lab Vacuum Tube	Univac 1103	EPSCO
Canadian National Research Council	G - 15	EPSCO
Grumman Aircraft	IBM 704	ADAGE
General Electric Philadelphia - MSD	IBM 7090	Homemade
NBC	SEAC	Homemade
North American Aircraft		PB
Bell Labs		
IBM - Owego	IBM 7090	PB
General Electric Syracuse	PB 250	GE
Applied Science Corporation of Princeton (ASCOP)		
PMR, Pt. Mugu	IBM 7090	PB
Aeronautic Research Associates of Princeton (ARAP)	LGP 30	
Westinghouse Research		
M.I.T.		
L.T.V. Chance Vought	PB 250	EAI
Martin Orlando	IBM 7090	EAI
McDonnell Aircraft	IBM 7090	McDonnell