REVIEWS

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Mathematical Comfort Food

Editor's Note: *At this point it is a cliche to say that the world has been chaotic lately. Most of us are dealing with many ramifications of the pandemic in both our personal and professional lives, not to mention increased anxiety related to the economy, racial tensions, and (as of this writing) the upcoming elections in the United States. I am hearing more and more that rather than new experiences people are revisiting movies, books, and television that have brought them happiness in the past—I know I am not the only person who rewatched the entire run of* Cheers *over the summer. So for this month's column, I thought that, rather than provide an in-depth review of a new monograph, I would ask a number of members of our community about some of the "mathematical comfort food" that they have turned to or that they might recommend people seek out. If you have suggestions for a future edition of this column, email me at the address below.*

Math for me and my childhood friends meant an endless source of puzzles. Growing up in a residential part of Budapest with little car traffic at the time, we loved posing questions to each other by writing and drawing on the road. While I remained infatuated with numbers and shapes through my teenage years, at school I came to fear that math is more about long calculations and memorizing formulas such as trigonometric identities.

So entering Eötvös University after high school, I was delighted to find myself in Edit Gyarmati and Róbert Freud's number theory course where the wonders of math came back into my life. The textbook we used was written by Gyarmati based on her lecture notes for Paul Turán's course, and I simply loved it. It was written with exceptional clarity, yet it was an exciting and far-reaching page-turner. Many years later, this Gyarmati–Turán textbook was revised, updated, and extended by Gyarmati and Freud, and now this book is translated to English by Freud and is published by the MAA/AMS [3].

I think the book is not only the best book on number theory, but the best textbook I have ever seen. Beginning students can gain a solid foundation in number theory, advanced students can challenge themselves with the often deep and always delightful exercises, and everyone, including experts in the field, can discover new topics or attain a better understanding of familiar ones. As with masterpieces in music and literature, one gets more out of it with each additional visit. I believe others will appreciate and enjoy this book just as much as I do.

— Béla Bajnok, Gettysburg College. Director of MAA Competitions

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The Symmetries of Things by John H. Conway, Heidi Burgiel, and Chaim Goodman-Strauss [1] is a lavishly illustrated treat. The authors have meticulously thought through each choice, from language that welcomes a general audience, to notation that aligns naturally with thinking, to color coding that aids understanding. This book is not only an incredible gift for students and visually oriented friends, it is an invaluable resource for researchers.

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Innumerable mathematical art projects have arisen as artists have been able to access the ideas in this book. The notions are explained in great depth and through multiple modalities. Each time I think I have come across an idea not present, I find that if I just look harder, there it is, illustrated, with algebraic explanation, and an accompanying chart! Regardless of the level of intellectual investment you intend to make with this book—whether it is simply perusing the illustrations, dipping into selected ideas, or truly studying full sections—I recommend this book as a stimulating addition to your library.

— Carolyn Yackel, Mercer University

During the summer of #BlackLivesMatter, I revisited the remarkable book *Black Mathematicians and Their Works* [**9**]. This 327-page volume from 1980 was edited by Virginia K. Newell, Joella H. Gipson, L. Waldo Rich, and Beauregard Stubblefield. The project was extremely ambitious at the time: the editors collected 23 research articles from black mathematicians, three articles from black mathematical educators, and several letters from Lee Lorch outlining some of the racist behavior by the professional societies in mathematics. The volume has some very serious research articles from some of the greatest black mathematicians: Albert Turner Bharucha-Reid, David Blackwell, William Claytor, Vivienne Malone-Mayes, Clarence Stephens, Walter Talbot, and J Ernest Wilkins. Never before—and never since!—has such a collection of work solely by black mathematicians been published.

Whenever I question whether black folk are making progress in these United States, I think of the articles in this volume, and those pioneers who continued to do math in the face of blatant racism.

— Edray Goins, Pomona College

This past summer and fall, as we have worked remotely on and off campus while trying to cope with unprecedented turmoil and change, I have repeatedly found myself drawn to movies that are familiar favorites. Perhaps it is because they feel warm and cosy, like a favorite pair of slippers or a comfy cardigan. Maybe it is because I know how the story line goes, so there is no stressful suspense, no uncomfortable surprises (I just don't have the bandwidth to deal with imaginary stress right now). But it could also be that they remind me of a more hopeful time in our country, when we felt as if our best days were ahead of us.

One of the movies that I have watched during this time is the movie *Hidden Figures*. [**7**] Why do I like *Hidden Figures*? Because success of women in STEM is one of my areas of expertise, I find the story hopeful in the cultural change it portrays. It reminds us that we have made some progress in nurturing the mathematical talent in our women and underrepresented students. I find it encouraging to see the growth of various characters in the movie with regard to their views on race and women. (I must add, that in light of the events of this past year, it is clear that change has happened at a snail's pace and that we have collectively not made nearly as much progress as we might have thought.) Of course, as a mathematician, I love the central role that mathematics plays in the movie. Some of it is dated. Anyone want to go back to bulky mechanical desktop calculators? But much of it is still relevant. For example, the movie does a great job of demonstrating the importance of critical thinking and problem solving, questioning and thinking outside the box, communication and interpersonal skills, persistence and effort, when doing mathematics. We try to drive these concepts home to our students every day. Finally, I think I am drawn to this movie because it reminds me of my childhood. I was born in the early 1960s in the state of Texas (home to NASA's Johnson Space Center), so I remember the excitement of the space race, the state and national pride we felt in getting to the moon, the way the clothing, cars, and houses all looked. So, if you are looking for a great movie to lift your spirits, fire up the popcorn machine, pop on your favorite sweatshirt, and turn on *Hidden Figures*.

Another movie I have enjoyed revisiting is *October Sky* [**5**]. This movie, released in 1999, tells the true story of Homer Hickham, Jr., from Coalwood, West Virginia, who was inspired by the launch of the original Sputnik spacecraft to become a NASA engineer. This inspiring story details how Homer and his friends overcame a lack of resources and know-how, in addition to the outright discouragement of their community, to do the impossible: build a rocket, win the national science fair, and secure a college scholarship. It was really the faith and the encouragement of their science teacher, Miss Riley, that helped them overcome all of these seemingly insurmountable odds. As I mentioned, this movie is a bit nostalgic for me. I grew up in a rural community similar to Coalwood a decade after the movie was set, and the look and feel of the movie is much like what I remember of my early school days. As a college professor myself (even though I am an engineering dean, I still teach freshman math), the movie reminds me of the difference that my belief in my students can make. Some years ago I started telling all of the students I teach that they are smart enough to make it if they will discipline themselves, ask questions, and work hard enough. You'd be surprised what kind of impact that can make. Students will often take their teacher's word as the gospel truth, much like Miss Riley's words inspired Homer and his friends to meet what was to them a formidable challenge.

I guess that at least part of me is hopeful that our nation will finally come together and rally around the goal of defeating COVID-19. I think that just like the space race inspired an entire generation to go into engineering, the pandemic very well may inspire an entire generation of students to pursue degrees in STEM fields, from medicine and engineering, to mathematical modeling, data analytics and visualization, and more. And maybe we can all be the Miss Rileys in this story. Maybe we can inspire our students to persist and succeed, despite the myriad challenges we are all facing right now, and do what may seem to many right now to be impossible and solve the grand challenges facing our world in the coming decades. That's a movie I'd like to watch.

— Jenna Carpenter, Campbell University

In October 1949, Alan Turing participated in a symposium organized by the philosophy department of the University of Manchester on "The Mind and the Computing Machine." The neurologist J. Z. Young discussed the physiology of nerve cells. Mathematician Max Newman debated the significance of Gödel's theorem with Michael Polanyi, who believed the theorem refuted the possibility of a mechanical explanation of thought. The brain surgeon Geoffrey Jefferson argued for the ineffability of mind, "Not until a computer can write a sonnet. . . ." You don't need to have been there to know the arguments made, you only need to read Alan Turing's "Computing Machinery and Intelligence." This is the paper in which Turing proposes what came to be called the Turing Test, but it begins by wittily, and occasionally snarkily, refuting all the arguments of the time arrayed against the idea of artificial intelligence, which you imagine him hearing at the Manchester symposium. Turing, of course, proposed a scientific resolution—you discover if a machine is intelligent in the same way you discover a person is intelligent, by having a conversation.

I'd read Turing's paper years ago, but I only discovered the context as a sort of extended *l'esprit de l'escalier* reaction to the symposium by reading Andrew Hodges's magisterial biography *Alan Turing: The Enigma* [4]. The book is much more than just an accounting of Turing's life. Sure, you get the details of his upbringing and his family and his path through life. And you get detailed, beautifully clear explanations of all of his scientific accomplishments. But the real insight is psychological: Hodges claims that Turing's homosexuality and atheism fundamentally shaped his worldview and his approach to science. Turing had an incredible ability to penetrate to the deep simplicities underlying complex phenomena and to discard the common, often implicit, assumptions that keep the rest of us from seeing that core. Hodges argues that Turing's outsider status enabled his scientific iconoclasm. You don't just learn the details of Turing's life and work here, you come to deeply appreciate his motivations and psyche and how those informed his scientific work and, ultimately, his tragic death. This is, by far, the most penetrating and satisfying mathematical biography I have ever read.

— Stephen Kennedy, Carleton College. Acquisitions Editor, MAA Publications

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Dasa Severova's book called *Origami Journey: Into the Fascinating World of Geometric Origami* [11] is itself a work of art. The book begins with a brief introduction aimed at those who might be new to origami. The author discusses the importance of paper when it comes to folding and describes her preferred types. She even recommends that readers participate in a papermaking workshop. Dasa Severova also gives tips and tricks for the readers, including using a sufficiently large paper according to one's folding experience. The author discusses the symbols that she uses throughout her book, as well as some useful tools for folding. The book is divided in three chapters: "Square Symmetry," "Let's Create Volume," and "Plenty in One Sheet." For example, in "Square Symmetry," the models are made from several identical units, called modules, that are joined together to create stars of flowers. This branch of paperfolding is called modular origami. These particular designs start with a square and use folds that bisect an angle. For this reason, they are strongly connected to square symmetry, which implies that they require 4, 8, or 16 modules if we would like the final design to be flat. All the origami models explained by the author are genuinely beautiful. The folding patterns presented in the book are clear and easy to follow. In particular, "Lonely Flower" is my favorite origami design in the book since it is simple and lovely, both mathematically and artistically.

— Jeanette Shakalli, Panamanian Foundation for the Promotion of Mathematics (FUNDAPROMAT)

I'm a knitter, not a crocheter, but my favorite book about mathematics and fiber arts is Daina Taimina's *Crocheting Adventures With Hyperbolic Planes* [12]. Taimina explains why negative curvature and crochet technique are suited to each other. Her models of hyperbolic planes are frilly and beautiful, like coral reef creatures. Taimina's models are also mathematically powerful: they form efficient illustrations of deep geometric ideas. I'm a particular fan of the transforming helicoid, which converts to a catenoid in a glorious isometry, sending geodesics to geodesics. As an algebraic geometer, I'm also delighted by Taimina's realization of Klein's quartic curve, which makes a special appearance in the second edition.

— Ursula Whitcher, AMS Mathematical Reviews

My nine-year-old was struggling with multiplication in her remote-instruction classes. Ideas about factors, arrays, and partial products flew across the Zoom window—and right over her head. I was struggling to help her and keep it fun.

Enter the stylish board game *Prime Climb* [10] from mathematician and TED speaker Dan Finkel and friends. The game features gorgeous visuals showcasing the prime decomposition of the first 101 natural numbers, numerical thinking to keep her practicing, and complex strategy that keeps me pondering my options. *Prime Climb* brings a little extra fun for both of us, and more mathematical understanding for Ellie.

— Dave Kung, St Mary's College of Maryland. Director, Project NExT

As a faculty member at a community college, I teach students taking math in the first two years of college. Some of these students do not have much exposure to the excitement and history of mathematics, so I enjoy bringing a little history into the classroom about the origins of mathematical topics. I find reading books on mathematical concepts enlightens me and gives me the knowledge to share with my students. One such book is *An Imaginary Tale: The Story of* $\sqrt{-1}$ by Paul Nahin [8]. This book starts with how mathematicians nearly 2000 years ago overlooked imaginary numbers. It then describes the roots of the cubic equation and how analyzing their values forced mathematicians to confront the square root of negative one. After that, the book goes into the geometry of complex numbers. The author describes how famous and not-so-famous mathematicians over the ages have worked on problems whose solutions involve the square root of negative one, and how the idea of imaginary numbers evolved.

Later in the book, the reader will be exposed to uses of complex numbers and the beginning of complex function theory. The works of many mathematicians—such as Cauchy, Euler, Gauss, Descartes, and John Bernoulli—are highlighted in the book. This allows the reader to humanize the development of the theory of complex numbers. The book is written at a level so that mathematicians will not find the book too pedestrian. There are appendices that explain a few mathematical concepts for non-mathematicians and mathematicians alike to enjoy. I suggest reading this book to learn more about the history of the square root of negative one. Then share with others so they can see the wonder of mathematics.

— Kathryn Kozak, Coconino Community College. President, AMATYC

The mathematical book that is my best fit in the "comfort food" category is *A Topological Picturebook* by George Francis [2]. When I was young and enamored with, but untrained in, mathematics, I used to browse through the stacks looking for books with nice pictures. I would then try to reverse-engineer the mathematics from the diagrams. This led to my being drawn to topology, the subject of Francis's text. What is unique about this picturebook is its emphasis on physical, practical drawing skills, in the service of elucidating mathematical ideas. One can read it as a novice or an expert in topology, with equal enjoyment. It's a rare and beautiful book that should be seen by every mathematician who wishes to communicate clearly with chalk.

— Robert Ghrist, University of Pennsylvania

My advanced math classes in college followed a standard pattern: in the beginning of the semester were the definitions, then came the lemmas, then the theorems, culminating at the end of the semester with the big proofs and then, if there was time, maybe some applications along with the much-despised "heuristics." And not a counterexample to be found. These were theorems, after all. A theorem is true and so has no counterexamples, right?

It was only in my senior year that I learned the proper order of mathematical reasoning: first the problem, then the theorem, then the proof, with the definition at the very end. The definitions come at the end because they represent the minimal conditions under which the theorem is true. The statement of the theorem itself changes as the proof and definitions develop. And, just as a country is defined by its borders, a theorem is bounded by its counterexamples, which are duals to its definitions.

I gained this perspective by reading *Proofs and Refutations* [6], a book taken from the Ph.D. thesis of the great philosopher Imre Lakatos. Lakatos's later work was in the philosophy of science, where his synthesis of the ideas of Karl Popper and Thomas Kuhn led to a sophisticated falsificationist model of scientific practice which influenced generations of social scientists. *Proofs and Refutations* finds a similar empirical sprit within mathematics.

The physicist Eugene Wigner wrote about "the unreasonable effectiveness of mathematics in the natural sciences." Flipping this around, *Proofs and Refutations* discusses the effectiveness of scientific inquiry in mathematical research, an idea which is commonplace in our modern era of computer experimentation but was controversial in 1964, when the book was published, or even twenty years after that, when I was taking all those conventionally constructed math classes.

In his book, Lakatos develops his ideas through a development of Euler's formula of the faces, edges, and vertices of polyhedra. It turns out this theorem has lots of counterexamples. One at a time Lakatos brings these up, in the setting of a fanciful conversation among hypothetical mathematics students, following the ideas of mathematicians working on this problem in the 1800s; these hypothetical students manage to rescue the theorem from each counterexample, but at an increasing cost. As a mathematics student, I found the story dramatic, even gripping, and when I was done, I had a new view of mathematics, an understanding I wish I'd had before taking all those courses—although I might not have been ready for it then.

As an applied statistician, I'm more of a user than a developer of mathematics—in my career, I've proven only two results that I'd dignify with the title "theorem," and one of them turned out to be false. But the same alternation of proof and counterexample arises for me when developing statistical methods and applying them to live problems.

– Andrew Gelman, Columbia University

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