Expanding psychological theory using system analogies

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Abstract

Background. System analogies are useful in that they facilitate our ability to understand new domains in terms of those that are already familiar. This powerful approach was employed early on by James Clerk Maxwell to show how particular ideal physical systems could be represented by the same class of mathematical equations despite having differing physical manifestations (mechanical, electrical, thermal, etc.). In physics, the coefficients of the model equations have real physical meanings, such as velocity or electric voltage, and the analogies speak to which variables play a particular mathematical role in a given physical domain. Established physical system analogies are still routinely used for pedagogical purposes, but here, analogy is employed to advance our understanding of a more newly discovered, less well-understood system, which is in the domain of psychological science. Specifically, the phenomenon of associative learning, which stems from the pioneering observations of Ivan Pavlov, is readdressed here with the tool of system analogy.

Advances. This paper recognizes that associative learning can be modeled with a specific type of equation (a first-order ordinary differential equation that fits the general solution), wherein the coefficients themselves have a real psychological meaning. By comparison to analogous systems in well-defined physical domains, a lawful relationship is posited using the psychological terms that correspond with the coefficients of the associative learning equation: (1) *for any observed event, the amount of attention paid to any event feature is proportional to the salience of the event feature and inversely proportional to the degree that the event is unsurprising*. The proportionality of salience and surprise follows the structure of Newton's second law, and would therefore be a special case of the law of conservation of energy. A second theoretical claim is also posited: (2) *the degree of surprise associated with any event feature that repeats itself always decreases*. This statement is analogous to the second law of thermodynamics, which accounts for the irreversibility of experience. Given this second statement, it follows that the variable of surprise can be minimized by repeating the same behaviors.

Outlook. This work draws a connection between classic physical variables and classic psychological variables, painting a unique theoretical bridge between psychology and the hard sciences by synthesizing the work of Maxwell and Pavlov. The framework described here uniquely formally defines psychological variables with respect to themselves rather than tying them to a specific operationalization, providing a theoretical approach that has the potential to generate useful and convenient measurement strategies for the field of psychology more broadly. Further, this work is a proof-of-concept that psychological processes can be modeled using equivalent circuits. This modeling strategy could be expanded on to simulate more complex behaviors in artificial agents.

“Recognition of the formal analogy between two systems of ideas leads to a knowledge of both, more profound than could be obtained by studying each system separately.” (Maxwell, 1870)

“What is mass?...Masses are coefficients which it is found convenient to introduce into calculations.” (Poincaré, 1905)

A number of reports have posited that only about half of the experiments conducted in psychology experiments yield results that can be replicated by other laboratories. In turn, a “replication crisis” has ensued, unfortunately causing many people in the general public to doubt whether or not the entire field of psychology can be considered scientific at all. Looking closely, we can see that the crisis itself stems from an underlying belief that if we somehow just *knew* which experimental results from psychology replicated, that this magical list of “true” facts would somehow appease our deep rooted desire to explain the way the mind works. In reality, replication is only at the very beginning of scientific endeavor. How many times did the planets have to traverse the sky before we finally realized a satisfying model? It took Kepler his entire lifetime, even as he was standing on the shoulders of Copernicus and Tycho.

Despite the perception of a crisis, there are myriad replicable effects within the field of psychology that demand an explanation, it seems unnecessary to spend much time lamenting those findings that do not or probably will not replicate. Take for example, that large-scale brain networks can be characterized from people doing nothing but mind wandering in an fMRI scanner (i.e., “resting state” data). The observed effects are highly replicable and the networks seem to map on to known psychological functions (Buckner, Andrews-Hanna, & Schachter, 2008), yet there remains some controversy over why the data look the same in every sample and what they mean. The fact that we see a pattern that replicates tells us nothing more than that there is a pattern that repeats itself. A mountain of work still lies ahead when we ask: how can we develop a scientific model of the pattern?

To answer this, it is perhaps important to first recognize that in our initial example, Kepler’s model of the solar system is not an *explanation* of the observed data (the data being the actual phase portraits produced by planetary motion in the night sky)—rather, his model is an accurate *formalization* of the data. To this day, we still do not have a philosophical explanation for why the planets behave the way they do, we simply know how to predict their behavior very precisely. The knowledge we gained from Kepler’s model became more general when Newton noted that the behavior of the planets represented a solution to a much more general “two-body problem” in physics. That is, any approximately isolated system of two masses will exhibit behavior that is approximately determined by Newton’s laws.

Consider that simply knowing about the general two-body problem and its solution represents a type of scientific knowledge that is an important deviation from the type of knowledge generated by what is called “null hypothesis statistical testing” (NHST), the latter type of knowledge being the basis of most psychological research, as well as being conceptually tied to the replication crisis mentioned in the opening paragraph. NHST considers the probability that some observed data structure is produced by random chance. Let’s ask ourselves: what are the chances that all isolated two-body systems of masses in the known universe can be solved by Newton’s ordinary differential equations, as opposed to some other more complicated equations? The chances are obviously zero. This is why Newton could “feign no hypotheses” with respect to his laws. We have no means of proving or falsifying the statement that a massive object in motion will remain in motion unless acted on by an outside force—if an object had no forces acting on it, how could we possibly know? The law is unfalsifiable, meaning it is outside the reach of the null hypothesis.

Curiously, the field of psychological science, being entrenched in the null hypothesis paradigm, almost unanimously opposes unfalsifiable statements as being unscientific (e.g., Meehl, 1978; in the footsteps of Popper). In contrast, the argument here, is that unfalsifiable statements must be allowed as long as they are convenient. Arguably, unfalisifiable laws are essential to scientific explanation beyond NHST, as NHST can only reveal isolated distinctions, which might lack general utility even if they can be replicated. How might the field of psychology construct its own legitimate unfalsifiable statements? This paper suggest one way of going about this, positing theoretical statements about psychological variables that have an unfalsifiable form, followed by a discussion of how this approach can potentially advance (a) measurement of psychological variables, and (b) simulation of behavior in artificial agents.

Background.

The field of psychological science as we know it today emerged at the same time that the field of physics was transitioning from classical mechanics to quantum mechanics, which was around the beginning of the twentieth century. Now, over a hundred years later, some universal psychological principles have been unpacked, including Fechner's law of just noticeable stimulus discrimination (1860/1912) and Shepard's law of stimulus generalizability (1987). Scientific progress unfolds at a painstaking rate: over a century had to pass before Newton recognized that Kepler's model of planetary motion was a specific example of the general two-body problem, and even more centuries passed after that before Poincaré would finally solve the three-body problem that was left open by Newton[1].

Nonetheless, the rate of progress for the adolescent scientific field of psychology can perhaps be accelerated if we learn by example. Although psychology is a unique science with its own unique problems of theory and method, we can still benefit from tapping into some of the more fundamental principles that other scientific domains have worked so hard to uncover. This approach can be implemented literally through the use of scientific analogy, which was most elegantly demonstrated in the timeless work of Maxwell[2,3]:

> "The most obvious case is that in which we learn that a certain system of quantities in a new science stand to one another in the same mathematical relations as a certain other system in an old science, which has already been reduced to a mathematical form...[by detecting] the analogy between any system of quantities presented to us and other systems of quantities in known sciences, we should lose no time in availing ourselves of the mathematical labours of those who had already solved problems essentially the same." (Maxwell, 1869)

To apply the technique of scientific analogy to psychology, we will take as our focus an observed phenomenon that has been replicated in thousands of psychological experiments, namely, the phases of associative learning. The discovery of the two major phases of associative learning, routinely referred to as (1) "conditioning" and (2) "extinction", can be traced back to

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[1] Poetically, Newton called the "three-body problem" the "problem of the moon". Poincaré (1905)

[2] "We all know that it was Maxwell who connected by a slender tie two branches of physics—optics and electricity—until then unsuspected of having anything in common." Poincaré (1905), p. 237

[3] "The equations of electrical circuit theory may be based on Maxwell's dynamical theory in which the currents play the role of velocities. Expressions for the kinetic energy, potential energy, and dissipation are deducible from general dynamic equations. In other words, an electrical circuit may be considered [more generally] to be a vibrating system." Olson (1943), p. 1

the groundbreaking observations of Ivan Pavlov (1927), and hence the phenomenon is sometimes referred to as “Pavlovian conditioning”. During the conditioning phase, a motivationally-neutral (conditionable) stimulus, such as a brief tone or light flash, is displayed by an experimenter to signal to a subject the occurrence of an unconditionably motivating stimulus, such as an aversive shock or appetitive food. The psychological association between the tone (conditionable stimulus, or CS) and the shock (unconditionable stimulus, or US), which manifests in the observable behavior and physiological responses of the subject, increases according to a saturating exponential that asymptotes at some maximum value. This maximum value, as well as the amount of time it takes to get there, both depend on psychological properties of the CS and the US. Following the conditioning phase, an extinction phase can be induced by presenting the tone in the absence of the reinforcing shock. In this second phase, the association between the CS and the US, again as measured by observable behavior and physiological response, decreases according to a decaying exponential.

The mathematical structure of the associative learning phenomenon implies that it can be modeled with a first-order ordinary differential equation that fits the general solution. Indeed, several models of this form have been proposed (Kendler, 1971; Rescorla & Wagner, 1972; Mackintosh, 1975; Pearce & Hall, 1980). Most of these models define the asymptote of the equation with a single term (λ) that is related to the US, but Konorski’s summary of Pavlov’s work suggests that this asymptote value is determined by both the CS and the US (1948), not just the US. Only Tracy Kendler’s model defines the asymptote in this way (1971). However, Kendler’s model underspecifies the rate parameter as being only tied to the CS, whereas other models define the learning rate as being tied to qualities of both the CS and the US (e.g., Rescorla & Wagner, 1972). In an attempt to synthesize these various existing models, equation (1) is proposed as a revised mathematical model of associative learning:

$$(1) \qquad \frac{dV}{dt} = \alpha_C \beta_U (\alpha_C \beta_U - V)$$

Equation (1) is a first-order ordinary differential equation, wherein the change in associative value over time (dV/dt) is a function of the current associative value (V). Here, associative value is defined as the psychological connection of a CS with a US, which can be measured via overt behavioral and physiological response patterns that correspond to experimentally controlled CS-US contingencies in time. In this equation, α_C is defined as the amount of attention paid to the CS, and β_U is defined as the salience of the US. Note that in this equation, the attention paid to the CS (α_C) is included in both the rate parameter and the asymptote. This is in contrast to most previous models (with the exception of Kendler, 1971), which have tended to define the asymptote with a single term λ, which traditionally has been labeled only literally as the maximum associative value, rather than being attached to a psychological construct (see Pearce & Bouton, 2001 for a review).

Whether or not equation 1 is “correct” is not the point of this paper (it is probably not correct). Rather, there are two points: (1) *identifying the optimal parameters for the associative learning equation is an open question in the field of psychological science*, and (2) *identifying these parameters will provide us the opportunity to generate mathematical definitions of attention and salience*. Furthermore, once the parameters of this equation are precisely agreed upon, we will be able to identify the deeper relationships between the parameters themselves, which is discussed in the following section.

A Formal Analogy.

Due to the mathematical structure of equation 1 (regardless of the particular parameters proposed here), the system underlying the phenomenon of associative learning can be classified as being one of several important linear dynamic systems. That is, there are ideal systems in various physical domains (mechanics, electronics, heat) that exhibit behavior with this same elegant mathematical structure (Figure 1). Drawing an analogy with these systems can allow us to see into a deeper level of the intriguing phenomenon of associative learning.

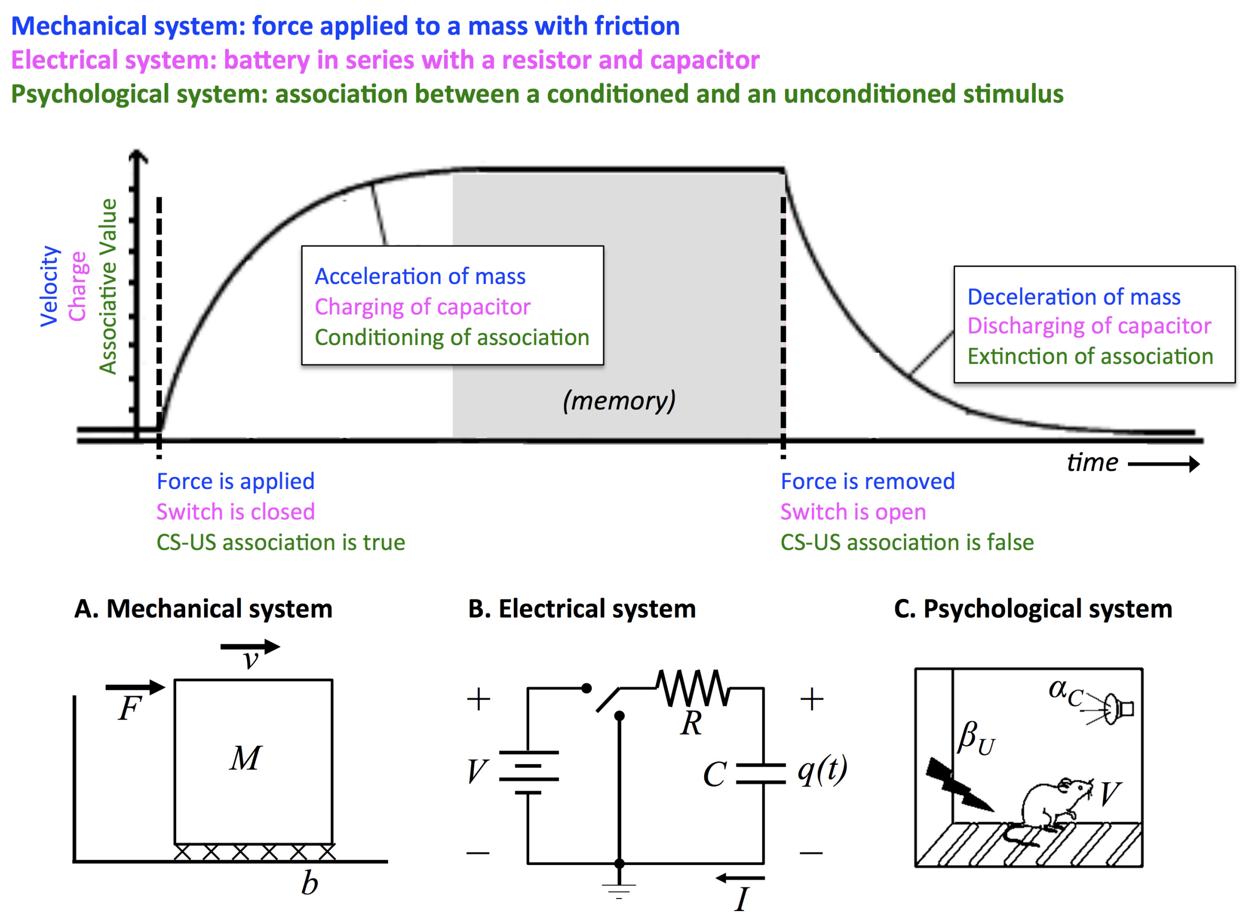

Figure 1. Analogous mechanical (blue, A), electrical (magenta, B), and psychological (green, C) systems that can all be described by first-order ordinary differential equations.

Importantly, first note that the behavior of the ideal mechanical (blue, A) and electrical (magenta, B) systems outlined in Figure 1 can both described by differential equations that are constructed from more basic underlying principles. For example, the differential equation describing the behavior of an electrical circuit that has a battery in series with a resistor and capacitor (a classic RC circuit, Figure 1B) can be derived using Ohm's Law, Kirchoff's Laws, and the lawful relationship between voltage and capacitance. In other words, *by knowing the basic laws, one can predict what behavior this particular system of arranged components will produce*. In the mechanical circuit, basic principles lie at the foundation of that differential equation as well (i.e., Newton's laws).

The theoretical debate surrounding existing models of associative learning can help us see how the psychological system might be deconstructed in a similar fashion. That is, there has been some question as to whether the learning rate term should be defined by "attention" instead of "salience" (Mackintosh, 1975; Pearce & Hall, 1980). This is an exceedingly difficult theoretical question to answer, precisely because we have an intuition that attention and salience are not the same, but are somehow fundamentally related. These terms are difficult to define with a verbal, dictionary definition. Again, the crux of the theoretical work here is to figure out how to effectively label the coefficients of the mathematical description of the phenomenon as a way toward achieving mathematical as opposed to verbal definitions. This is important because equations can be empirically verified, whereas verbal definitions can only be intuitively agreed upon. Here, it is proposed that we have an intuition about the proportionality of salience and attention, which is *like* our intuition about the proportionality of force and acceleration. How can we separate acceleration from force? We cannot—they are inextricably related. Following this logic, a principle that is analogous to Newton's second law can be formulated using psychological variables:

(2) $\beta = \alpha\gamma$

where α = attention, β = salience, and γ = familiarity. From this mathematical statement comes the first claim proposed in the abstract: *the amount of attention paid to any event feature is proportional to the salience of the event feature and inversely proportional to the degree that the event is unsurprising*. These interrelated variables can in theory be applied to specific features of an event (e.g., the conditioned stimulus) or the event as a whole (e.g., the entire experimental trial). According to this theory, unsurprising events must be more salient in order to receive the same amount of attention as a surprising event. No causal direction is implied, as explicit ("top-down") attention to any event feature can makes that feature more salient (by definition here). The structure of equation (2) is such that it can be considered a specific case of the law of conservation of energy.

Beyond this, an additional statement can be proposed. Consider that for any specific action or observation, our experience tells us that the degree of surprise always decreases as familiarity increases with repetition. In turn, a principle that is analogous to the second law of thermodynamics can be proposed to highlight the directional nature of how surprise unfolds over time: *the surprise of any event feature that repeats itself always decreases*. In psychology there is no such thing as a truly repeated measure, because experience itself is irreversible (e.g., Luce, 1999), and even the degree of unpredictability is a value the brain learns to predict (Whalen, 2007). Repetition—that is, *familiarity*, or lack of surprise—is the key to all prediction. The first trial of an experiment is always more surprising to the participant than the second, no matter what the topic of investigation is. This principle is at work in all psychological studies that measure responses to repeated presentations of the same stimulus or type of stimulus.

Notably, this second claim is related to the recently coined "free energy" principle, which posits that the minimization of surprise (which is also the maximization of familiarity) is a goal of biological systems generally (Friston, 2010; Gershman, 2019). Here, there is no claim made about whether or not the minimization of surprise is optimal. The theoretical statement does, however, suggest that repeated behavior is a general mechanism that can minimize surprise.

Discussion.

What is the utility of making these kinds of theoretical claims? First, theoretical statements like this can advance measurement strategies. We all know how critical measurement is for science[4], and theoretical statements like these are what refine measurement strategies. The theoretical statements outlined here will hopefully service the development of more sophisticated measurement strategies in psychology, and this point is further elaborated on in this discussion.

Second, there is great convenience to be found in translating dynamic systems of any domain into the domain of electrical circuits (Olson, 1943). Here, we have seen how a simple psychological conditioning paradigm can be modeled with a classic RC circuit, opening the door to potentially designing equivalent circuits of more complex behaviors that have more parameters. This approach to formalizing complex behaviors could be useful for simulating particular behaviors in artificial agents, and an avenue for this future work is highlighted in the final remarks.

The relationship between measurement and theory. Without measurement, the whole institution of science could not exist, measurement is the very foundation of the method. Before delving into how the proposed theoretical statements might inform psychological measurement, it will be useful to first consider the generally elusive nature of measurement. Consider for a moment a familiar example in the physical domain—how is it possible for us to *know* that a thermometer is an effective measurement apparatus for temperature? By putting mercury into a glass cylinder, we *assume* that if the room is warmer than the mercury, heat will be transferred from the room to the mercury (i.e., we assume the second law of thermodynamics), and we *assume* the density of the mercury will change (i.e., we assume the density of liquid is lawfully proportional to temperature) until there is no difference in temperature between the room and the mercury, at which point we can take a temperature reading (on an ordinal[5] scale with arbitrary reference points related to the properties of water).

There are two things to consider here. First, note that the measurement apparatus itself is built using theoretical principles. No one would have even thought to put mercury inside a glass cylinder unless they had some sense of the basic theoretical properties of thermal dynamics. The density of mercury is not merely an operationalization of heat, it is a measurement that conveniently reflects the theoretical dynamics of heat. Heat, by definition, is a measured quantity and nothing more (Maxwell, 1872). There really is no measurement *of* heat—heat itself is the measured quantity[6]. When someone tries to "find a measurement of X", there is some clumsiness in the structure of this language that promotes a subtle misunderstanding about measurement. There is no substance beneath measurement: form is empty. In this sense, we can see that *the art of measurement involves designing convenient ways of observing natural principles in action*. Knowing (or assuming) what the natural principles are helps us identify robust measurement strategies. In other words, if we theorize that things operate in a certain way, then we can then identify convenient ways to measure those things.

Second, note that putting mercury inside a glass cylinder is not the only way to measure temperature—there are perhaps infinite ways. Temperature could be measured by referring to the

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[4] "When we say force is the cause of motion, we are talking metaphysics; and this definition, if we had to be content with it, would be absolutely fruitless, would lead to absolutely nothing. For a definition to be of any use it must tell us how to measure force; and that is quite sufficient, for it is by no means necessary to tell what force is in itself, nor whether it is the cause or the effect of motion." Poincaré (1905), p. 110

[5] "We have no means of estimating numerically the difference between two temperatures, so as to be able to assert that a certain temperature is halfway between two other temperatures." Maxwell (1872), p. 32

[6] "We have therefore a right to speak of heat as of a measurable quantity, and to treat it mathematically like other measurable quantities so long as it continues to exist as heat. We shall find, however, that we have no right to treat heat as a substance, for it may be transformed into something which is not heat, and is certainly not a substance at all, namely, mechanical work." Maxwell (1972), p. 7

changes of density in any liquid, or by noting subtle changes in the speed of sound, or by observing changes in electrical resistance. The density of mercury just happens to be convenient because its density undergoes large enough changes to be easily perceived by our visual system, and it remains a liquid within a practical temperature range. Heat is a "valid" theoretical construct because we see it influence many measurements (Cronbach & Meehl, 1955), and we have theories about the precise nature of these influences. Heat is never actually directly observed in any circumstance, changes in the density of liquid are observed. In theory, the same can be said for surprise, salience, and attention—there are many ways these variables can influence measurements and become observable through them. Lawful assumptions about the measured variables is a necessary condition for developing precise measurement.

Measuring surprise and attention. We can know that our measurements are valid if they align with the theoretical principles by which we define the measured variables. Due to the theorized decay of surprise with repetition, any measured feature (perhaps behavioral or physiological) that exhibits the theoretical shape of this decay following repetition could be taken as a measure of surprise. There are likely many such features and the most convenient or consistent feature could be selected. It is possible that such a measurement might be best obtained in some highly contrived way (analogous to putting mercury in a glass cylinder). The adequacy of the measurement approach depends on the "truth" (i.e., convenience) of the claim that surprise decreases with repetition.

Attention is already intuitively measured via reaction time to discrete, controlled stimuli (e.g., Posner, 1980), which would be akin to synchrony with a continuous, naturalistic stimuli (the latency of the synchrony representing a continuous reaction time). What is often understated, however, is that reaction time *is* mechanical velocity. This ties the psychological variable to a classical mechanical variable, consistent with the analogies provided. When we express ourselves at each moment through physiological motion, this motion is continuous and differentiable, because it occurs through a physical body that obeys classical mechanics, fluid dynamics, thermodynamics, etc. In theory, psychological variables can be defined by the parameters of the physical expression (i.e., bodily motion) in the physical domain, just as in the associative learning example highlighted here.

Final remarks & future work. All behavior is part of a natural circuit. Stimulation comes in through breathing, eating, and the special senses, and the nervous system transforms the incoming signal into behavioral and physiological output, the status of which is always fed back to the system. Translating elements of this signal processing flow into terms of electrical circuits could be a promising modeling strategy for understanding psychological constructs, if circuit parameters can be tied to psychological variables as demonstrated in the example here.

Importantly, the variables used in equation 2 (attention, salience, familiarity) will influence circuits at smaller levels of the system, and this relationship could be further specified in future work. For example, note that at the outer edges of the sensory mechanisms, we find analog sampling filters that can be modeled with the same RC circuit as the global associative learning process highlighted here. That is, a single cilia of an inner ear hair cell *is* an analog sampling filter, operating as an ideal bandpass filter that resonates at a single frequency. In turn, the operation of an inner ear cilia can be "artificially" simulated using classic RC circuit components (i.e., a switching means and a memory means; McCullough, 2000a, 2000b, 2001), which pass a single frequency, consistent with the Nyquist sampling theorem. There are only two control parameters of an analog sampling filter: sampling frequency and sampling time. The control parameters of an analog sampling processor are almost identical: sampling frequency, sampling time, and signal detection. The latter, signal detection, is useful to initiate feed-forward signal processing functions or initiate further signal processing feedback for control and correction in real-time (i.e., in the moment), hence enabling an analog sampling processor to electrically model the variables used in equation 2 (attention, salience, familiarity).

In general, *the nervous system filters input that is analog*, and there is no analog to digital conversion step. Rather, the filtered input is further processed according to task demands that change in real-time according to psychological parameters. That is, tasks are selected by the organism in real-time to maintain psychological states such as safety, satiety, etc. In this sense, the human body *is* an analog sampling processor, which uses many analog sampling filters to sample (attend/switch) and hold (remember) data in real-time. Analog sampling filters are pervasive at many levels of biological systems, and even the neuron membrane itself is effectively modeled as an RC circuit (e.g., see Koester & Siegelbaum's chapter in Kandel, Schwartz, & Jessell, 2013).

In closing, one an assess that artificial agents designed to continuously sample the environment would likely benefit from the efficiency inherent to analog sampling processors, as the nervous system itself benefits from this efficiency, and the proposed framework offers a strategy for modeling the sampling process in terms of psychological variables. Most input can be ignored, so it is convenient to have an input that consists of tiny filtered fragments of signal (e.g., single frequencies), that can be flexibly processed according to goals. Psychological parameters (e.g., salience) determine which aspects of sensory input are further processed, in order to effectively guide behavior while minimizing unnecessary or undesirable processing costs. Considering these final points, along with the other principles argued in this paper, circuit theory models of biological motion (associative learning being the simplest example) could provide a promising framework for psychology, including simulating psychological processes in artificial agents.

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