All Models are Right

... most are useless

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George Box's Quote

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George Box's Quote

> *"All Models are Wrong, some are useful"*

This quote is useful ... but wrong.

Here is an extended quote:

... The fact that the polynomial is an approximation does not necessarily detract from its usefulness because all models are approximations. Essentially, all models are wrong, but some are useful. However, the approximate nature of the model must always be borne in mind."

From the book: Empirical Model-Building and Response Surfaces (1987, p 424), by Box and Draper.

Models are Approximations – Can approximations be Wrong?

$$\pi = 3.14$$

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Models are Approximations – Can approximations be Wrong?

$\pi = 3.14$ This is WRONG

$\pi \approx 3.14$ is not wrong

Stay Positive

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When teaching, why focus on the negative aspect of Box's quote:

> "Ok class, today I will introduce regression models. Oh, and by the way, all these models are wrong."

Instead:

> "Ok class, today we will introduce regression models which can be very useful approximations to the truth."

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Fallacy of Reification: *When an abstraction (the model) is treated as if it were a real concrete entity.*

- The fallacy of reification is committed over and over, even by statisticians, who believe a particular model represents the truth ... instead of an approximation.
- The model is not wrong but treating the model as the absolute truth (i.e. reification) is wrong.

A Dress, A suit, A Model

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"Does this model make me look fat?"

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"No dear"

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$$y = \beta_0 + \beta_1 x + \epsilon, .$$

Saying:
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Saying:
"Assume ϵ is approximately normal" will often be accurate.

A Quote

Paul Velleman writes:

> *"A model for data, no matter how elegant or correctly derived, must be discarded or revised if it does not fit the data or when new or better data are found and it fails to fit them."*

From "Truth, Damn Truth, and Statistics" in the *Journal of Statistical Education*, 2008.

Velleman's quote is useful ... but not always

Newton's 2nd Law of Motion $F = ma$ hasn't been discarded...

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$$F = \frac{d\{mv\}}{dt}.$$

$F = ma$ is still a useful approximation...as long as you don't go too fast.

Cylinder-Shaped Soda Can Example

Model soda volume as a function of height of soda in can

$$\text{Volume} = \beta_0 + \beta_1 \text{height} + \epsilon.$$

Then $\beta_0 = 0$ and $\beta_1 = \pi r^2$.

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Thad: "No, it is correct."

Soda Cans continued ...

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The full model Volume $= \beta_0 + \beta_1$height $+ \epsilon$ is useless if height of the soda in the can was not measured.

The reduced model $y = \beta_0 + \epsilon$ is equivalent to $y = \mu + \epsilon$ ... which is a correct model.

Parameters – A Source of Confusion

In the soda can example, the same symbol β_0 is being used to represent two different parameters.

Question: What is a parameter?

True Model, Approximation Model

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Question: But what is $\boldsymbol{\alpha}^*$?

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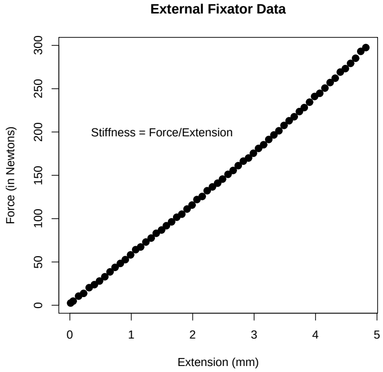
Question: But what is $\boldsymbol{\alpha}^*$?

$$\boldsymbol{\alpha}^* = \arg\max_{\boldsymbol{\alpha}} \int f(y; \boldsymbol{\theta}^*) \log(h(y; \boldsymbol{\alpha}))dy.$$

Example: External Fixator to hold a broken bone in place.

Thad: I'm going to use the slope of a straight line to estimate the stiffness of an external fixator.



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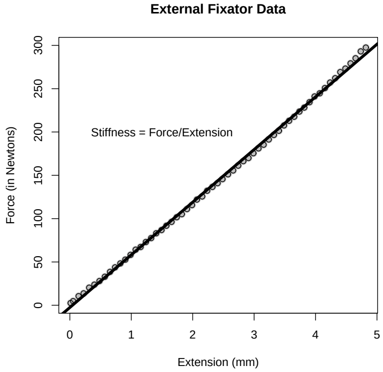

Regression

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Thad: “No, it is not wrong and quit calling me Shirley.”

Least Squares

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Then

$$\boldsymbol{\alpha}^* = \arg\min_{\boldsymbol{\alpha}} \int (f(x; \boldsymbol{\theta}^*) - \tilde{f}(x; \boldsymbol{\alpha}))^2 dF_x.$$

Illustration: A Straight-Line Approximation

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Extract the Linear Trend: $E[y|x] \approx \alpha_0 + \alpha_1 x$.

The least-squares criterion (for $x \sim U(0,1)$) gives

$$\alpha_0 = \mu_y - \alpha_1 \mu_x, \quad \text{and} \quad \alpha_1 = \sum_{j=0}^{\infty} \frac{6j\theta_j}{(j+2)(j+1)}.$$

Model body fat percentage as a function of weight

Predict Body Fat % Using Regression
(data from Johnson 1996, *JSE*)

Model body fat percentage as a function of weight: $\hat{y} = -9.99515 + 0.162\text{Wt}.$

Body Fat % continued ...

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Fellow Statistician: "Tarpey, your model is wrong...under-specified – there are other variables that also predict body fat percentage; your estimated slope will be biased. You need more predictors"

Multiple Regression

In multiple regression with two predictors x_1 and x_2 correlated to each other and to y:

$$\begin{aligned}\textbf{Full Model} &: \quad y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon. \\ \textbf{Reduced Model} &: \quad y = \beta_0 + \beta_1 x_1 + \epsilon.\end{aligned}$$

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We may drop x_2 for the sake of model parsimony or because x_2 does not appear significant.

Question: What is wrong with what I have written here?

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- The coefficient β_1 in the full model is never the same as β_1 in the reduced model unless $\beta_2 = 0$.

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- β_2 in the full model equals zero if and only if
$$\text{cor}(x_1, x_2) = \frac{\text{cor}(x_2, y)}{\text{cor}(x_1, y)}.$$
- Hence, β_2 cannot be zero if $\text{cor}(x_2, y) > \text{cor}(x_1, y)$.

Model Under-specification

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- In the model under-specification literature, $\hat{\beta}_1$ in the reduced model is called biased.
- According to this logic, $\hat{\beta}_1$ in a simple linear regression is always biased if there exists any other predictor more highly correlated with the response.

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Thad: No, the model is clearly right.

Coefficient Interpretation in Multiple Regression

The usual interpretation of a coefficient, say β_j of a predictor x_j, is that

> β_j represents the mean change in the response for a unit change in x_j *provided all other predictors are held constant.*

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- Body fat % will go down ...

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- Consider the population of men with some fixed abdomen circumference value.
- What happens to body fat percentage as the weights of men in this group increase?
- Body fat % will go down ... hence the negative coefficient.

Probability Models

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Basically, "...randomness is fundamentally incomplete information (Taleb, *Black Swan*, p 198).

Pick a Card ... any card

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Thad: 4/52 (I haven't seen the card).

Confidence Intervals

A quote from Devore and Peck's book *Statistics: The Exploration and Analysis of Data* (2005, p 373) regarding the 95% confidence interval for a proportion π:

> "... it is tempting to say there is a 'probability' of .95 that π is between .499 and .561. *Do not yield to this temptation!*...Any specific interval ... either includes π or it does not...We cannot make a chance statement concerning this particular interval."

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Compute a 95% confidence interval for μ.

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Compute a 95% confidence interval for μ.

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95% of all possible confidence intervals contain μ ... Chance has picked one of them for us...similar to picking a card from the deck.

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Question: What is the probability that my interval contains μ?

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For the Omniscient: 0 or 1

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Thad: If I don't know if μ is in the confidence interval... from my perspective, there is uncertainty; the probability cannot be 0 or 1. The correct probability model is conditional.

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With enough data, any imperfection in a model can be detected.

The temptation then is to say all models are wrong.

However, if we regard models as approximations to the truth, we could just as easily call all models right.

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Most of these will be useless ...

and perhaps a few will be useful.

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Models have served us very well ...

and also, at times, quite poorly.

Some quotes: Breiman 2001 *Statistical Science*

"...as data becomes more complex, the data models become more cumbersome and are losing the advantage of presenting a simple and clear picture of nature's mechanism (p 204)...

Unfortunately, our field has a vested interest in data models, come hell or high water (p 214)."

Some quotes: Taleb, *Black Swan*

"...the gains in our ability to model (and predict) the world may be dwarfed the increases in its complexity (p 136)"

A Final Quote:

Or, as Peter Norvig, Google's research director, says

> *Let's stop expecting to find a simple theory, and instead embrace complexity, and use as much data as well as we can to help define (or estimate) the complex models we need for these complex domains.*

* From http://norvig.com/fact-check.html. Note, Norvig was misquoted using a variation of the Box quote in: The End of Theory: The Data Deluge Makes the Scientific Method Obsolete" by Chris Anderson in Wired Magazine, 2008.