Two Simple Putting Models in Golf

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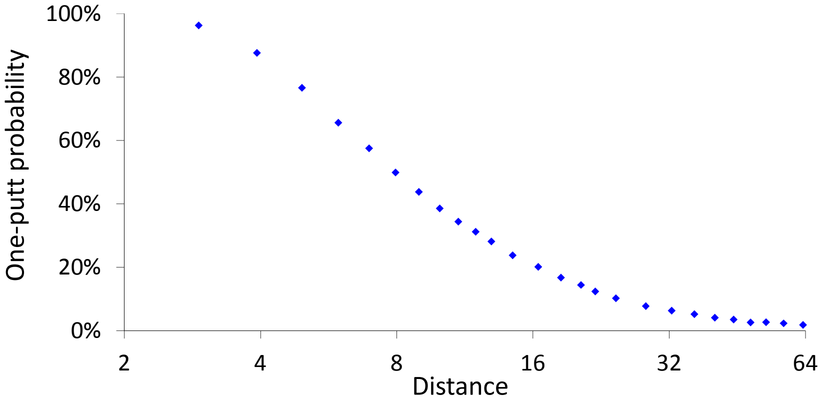
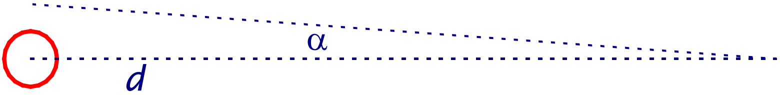
PGA Tour Putting Data



- PGA Tour data from 2016-2018 represents more than 1.2 million putts
- Standard errors range from 0.01% to 0.3% (not shown for clarity)
- Horizontal axis (initial putt distance, in feet) shown in log-scale for clarity

Gelman and Nolan Putting Model: Random Direction



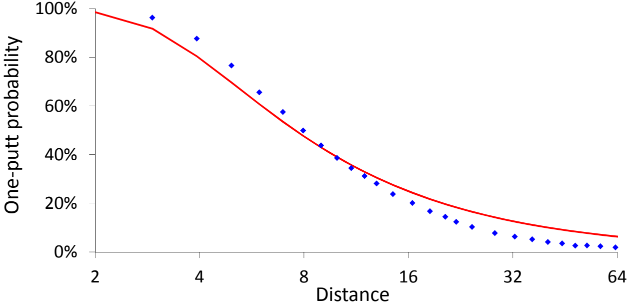
One-putt: $|\alpha| \le \alpha_c$, where d is the distance to the hole, r is the radius of the hole, and $\alpha_c = \tan^{-1}(r/d)$.

Suppose $\alpha \sim N(0, \sigma_\alpha^2)$. Then

$$\begin{aligned} P(\text{One-putt}) &= P(|\alpha| \le \alpha_c) = P(|Z| \le \alpha_c/\sigma_\alpha) \\ &= \Phi(\alpha_c/\sigma_\alpha) - \Phi(-\alpha_c/\sigma_\alpha) = 2\Phi(\alpha_c/\sigma_\alpha) - 1 \end{aligned}$$

Gelman and Nolan (2002)

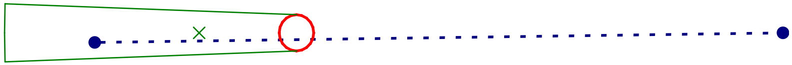
Gelman and Nolan Model: Fit to PGA Tour Data



Model: $\alpha \sim N(0, \sigma_\alpha^2)$. Choose σ_α to minimize the sum of squared differences between the model and the data. Optimal: $\sigma_\alpha = 2.00^\circ$ (RMSE: 4.6%)

- Model probability $\rightarrow 1$ as $d \rightarrow 0$ and $\rightarrow 0$ as $d \rightarrow \infty$
- Model is biased low for $d < 8$ and biased high for $d > 8$
- See `pga_tour_putt_data_models.xlsx` for details

Broadie Model: Random Distance and Direction

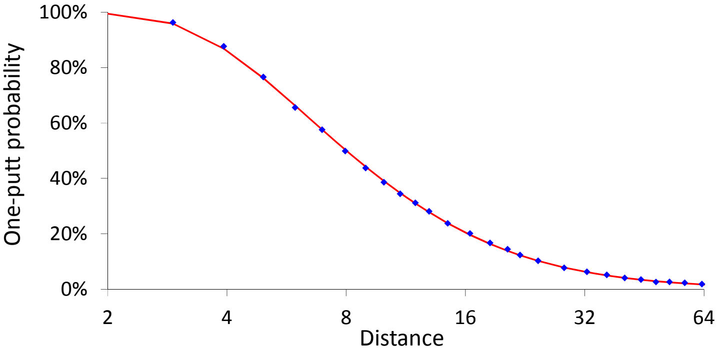


One-putt if endpoint in the *hole out region*: $|\alpha| \le \alpha_c$ and the putt distance, l, satisfies $d \le l \le d+3$.

Suppose $l = (d+1)(1+\sigma_d Z), Z \sim N(0,1)$, i.e., the target is one foot beyond the hole, σ_d is the fractional distance error and Z is independent of α.

$$
\begin{aligned}
P(\text{One-putt}) &= P(|\alpha| \le \alpha_c) P(d \le l \le d+3) \\
&= P(|\alpha| \le \alpha_c) P\left(\tfrac{-1}{\sigma_d(d+1)} \le Z \le \tfrac{2}{\sigma_d(d+1)}\right) \\
&= \left(2\Phi(\tfrac{\alpha_c}{\sigma_\alpha}) - 1\right)\left(\Phi(\tfrac{2}{\sigma_d(d+1)}) - \Phi(\tfrac{-1}{\sigma_d(d+1)})\right)
\end{aligned}
$$

Random Dis and Dir Model: Fit to PGA Tour Data



- Optimal parameters: $\sigma_\alpha = 1.69^\circ$ and $\sigma_d = 7.96\%$ (RMSE: 0.3%)
- Model fits well for all distances
- See `pga_tour_putt_data_models.xlsx` for details