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## Introduction

Using a Bayesian approach for making statistical inferences has been gaining popularity in recent years. Stan (<http://mc-stan.org>) is a Bayesian probabilistic programming language that implements an efficient Hamiltonian Monte Carlo method suitable for fitting larger and more complex models, and these capabilities are attracting more and more users, pharmacometricians in particular.

Currently, two hurdles have largely limited a broad application of Stan in pharmacometrics: 1) a steep learning curve for pharmacometricians to write PKPD model-specific C++-like Stan code; 2) no efficient solvers to work seamlessly with Stan's No-U-Turn Sampler (NUTS) for ordinary differential equations (ODEs) that are able to handle stiff ODE systems, often encountered in PKPD modeling.

Here we provide an R package called **PMXStan** to facilitate practical Bayesian PKPD modeling and simulation using Stan.

## PMXStan for Bayesian PKPD modeling

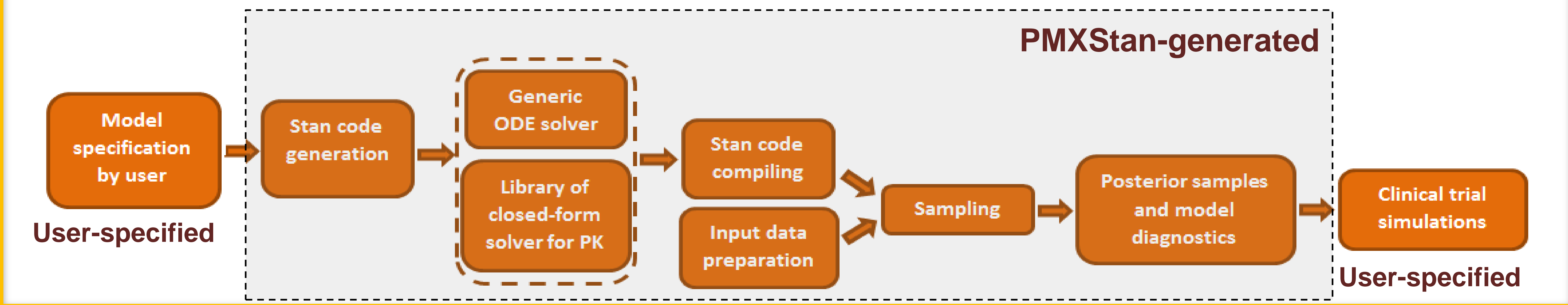
**PMXStan** helps pharmacometricians to focus more on PKPD model building and frees users from intimidating coding not commonly used in the pharmacometrics community. More specifically,

- 1) PMXStan automatically handles low-level technical details using a set of wrapper functions; and
- 2) PMXStan provides a NUTS compatible template LSODA solver to deal with stiff ODE systems.

Some advantages of using PMXStan include, but not limited to:

- With a few model specification statements defined by a user, PMXStan **generates model-specific ready-to-run Stan source code**, which is fully accessible and modifiable by the user.
- PMXStan uses data-conversion functions to **translate a conventional NONMEM dataset into a data list readable by Stan**, and provides convergence checks and model diagnostics.
- While **closed-form solutions are provided for PK models** when applicable in PMXStan, for a general PKPD model expressed as a set of ODEs, **the NUTS compatible template LSODA solver can be conveniently called**.

## Flow chart of model specification, compilation, execution, diagnostics, and simulations using PMXStan



```

1 m.type = startModelBuilding("PK")
2
3 # Model specification
4 input.specs = list(
5   m.path = "pk_cls_1",
6   d.type = "population",
7   m.type = m.type,
8   m.pk.struct = "2-cmpt",
9   m.pk.admin = "IV_infusion",
10  m.pk.param = "CL_V",
11  m.pk.solver = "closed_form",
12  m.datafile = "../datasets/popk_ivinfus_theo.csv"
13 )
14 # Proof checking
15 model.specs = checkModelSpecs(input.specs)
16
17 # Generate Stan source code for the specified model
18 stanfilename = generateStanCode(model.specs)
19
20 # Prepare input data for Stan
21 dat = prepareInputData(model.specs)
22
23 # Model fitting
24 fit = stan(file.path(model.specs$m.path,stanfilename),
25           data=dat, chains=1, iter=400)
26 # Print model fitting results
27 capture.output(print(fit,digits = 3, probs = c(0.025, 0.5, 0.975)),
28               file = file.path(model.specs$m.path, "summary.txt"))
29 # Save model specifications and fitting
30 save(model.specs, stanfilename, dat, fit,
31      file = file.path(model.specs$m.path, "model.info.RData"))
32
33 # Trace plots for parameters
34 pdf(file.path(model.specs$m.path,"trace.pdf"))
35 plotTraces(fit, model.specs)
36 dev.off()
37 # Goodness of fit
38 pdf(file.path(model.specs$m.path,"gof.pdf"))
39 plotGoF(fit, dat, model.specs)
40 dev.off()
    
```

## Model building, fitting, and diagnostics process for a population PK model

### Model specification by user

A 2-compartment population PK model with IV infusion, parameterized by clearance-volume, and solved by closed form solution

### Automated modeling process

- Stan code generation
- Data preparation
- Invoke Stan for model compiling and sampling

### Post processing

- Trace plots to check convergence
- Goodness-of-fit plots for model diagnostics

## Generic PKPD models in ODE form

- User provides a set of ODEs
- A customized solver ("ODE extension") is generated for the input ODE system
- System parameters are recognized and output for the convenience of model specification by users

```

> ode <- "
  C2 = centr/V;
  d/dt(depot) = -ka*depot;
  d/dt(centr) = ka*depot - ke*centr;
  d/dt(eff) = (1+Emax*C2/(C2+EC50))*Kin -
  Kout*eff;
"
> instant.stan.extension(ode)
A new ODE extension for Stan has been
created.
System parameters are: V ka ke Emax EC50 Kin
Kout
    
```

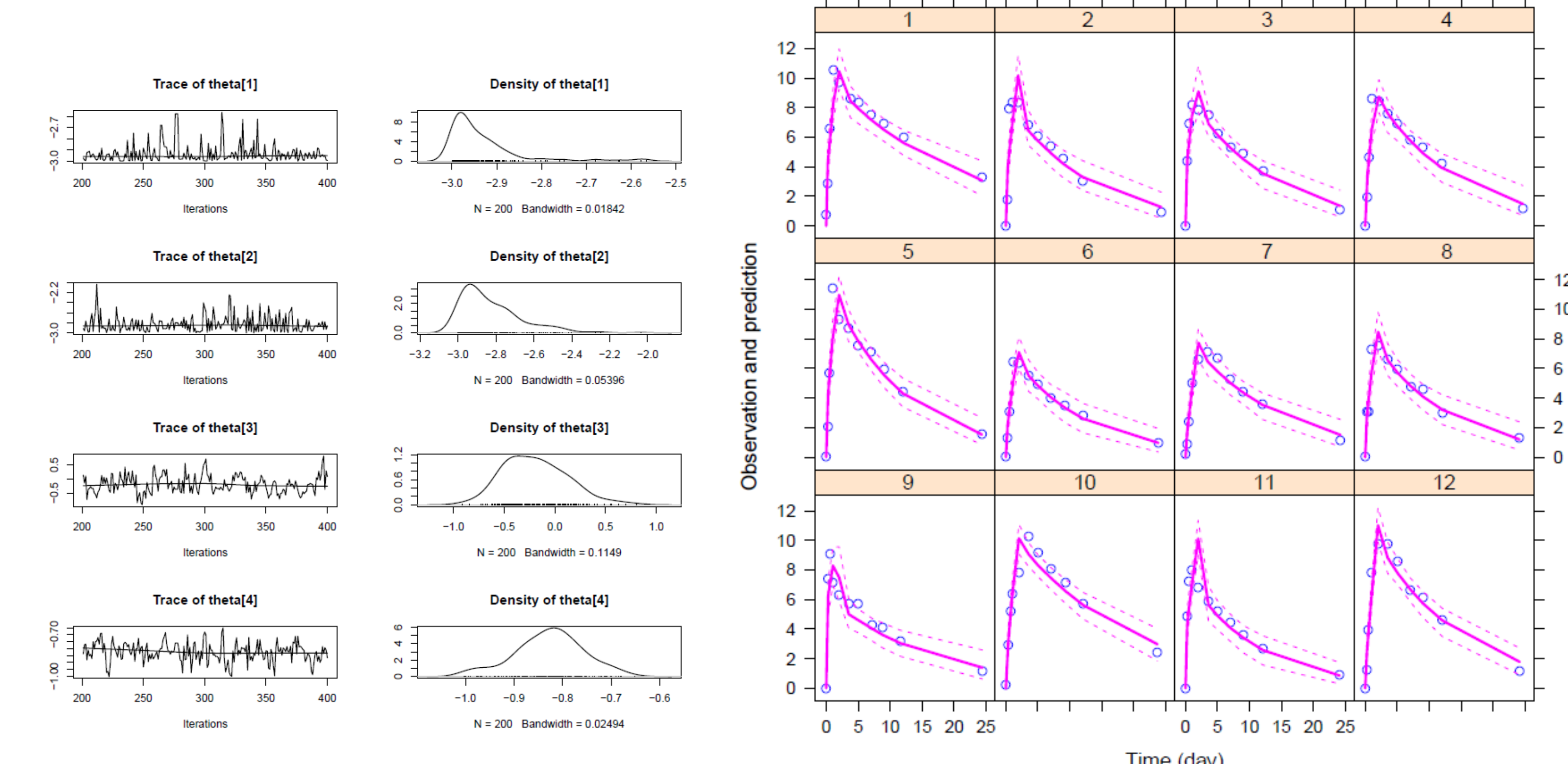
### Main features

- Written in C++ and highly efficient
- Handling complex dosing events of various routes and schedules
- Capacity to fit multiple endpoints simultaneously

Specification	Specification Variable	Options
<i>Common for both model types</i>		
Model type	m.type	PK   PKPD
File path for the model	m.path	User input path
Data type	d.type	individual   population
Drug administration	m.pk.admin	1st_order_abs   IV_bolus   IV_infusion
<i>For PK models only</i>		
PK model structure	m.pk.struct	1-cmpt   2-cmpt   3-cmpt
PK model parameterization	m.pk.param	CL_V   micro_rate
PK model solver	m.pk.solver	closed_form   ODE
<i>For PKPD models only</i>		
Index of observed state variable	m.obs.idx	An integer
Parameters to be estimated	m.theta	Choose from parameter list
Between-subject random effects	m.eta	Choose from m.theta
Parameters not to be estimated	m.const	Input values of constant parameters
Initial values of state variables	m.obs.init	Extract from data

```

data{
  int<lower=0> NSUB;
  int<lower=0> NOBS[NSUB];
  int<lower=0> NDOSE[NSUB];
  vector[sum(NOBS)] conc;
  ...
}
parameters{
  vector<lower=-5.0, upper=5.0>[4] theta;
  vector[4] eta[NSUB];
  ...
}
transformed parameters{
  ...
  for(i in 1:NSUB){
    ...
    g <- linear_cmpt_iv_infusion(...);
    ...
  }
}
model{
  for(k in 1:4){
    for(i in 1:NSUB)
      eta[i,k] ~ normal(0.,1.);
      theta[k] ~ normal(0.,1000.);
      ...
  }
  conc ~ normal(y_pred, sigma);
}
    
```



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