Misconception of Chance Processes in Basketball

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Abstract

Basketball players and fans commonly believe that players tend to shoot in streaks and that the chances of hitting a shot are greater following a hit than following a miss. Analyses of both professional and college basketball reveal that, contrary to common belief, the outcomes of successive shots are largely independent. The "detection" of streaks in random sequences is traced to an erroneous intuition that the law of large numbers applies to small samples as well.

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Intuitive conceptions of randomness do not conform to the laws of chance. People commonly believe that the essential characteristics of a chance process are represented in small as well as in large samples. Thus people expect even short sequences of heads and tails to reflect the fairness of a coin and contain roughly 50% heads and 50% tails. This conception of chance has been described as a "belief in the law of small numbers" according to which the law of large numbers applies to small samples as well (1). A locally representative sequence, however, deviates systematically from chance expectation: it contains too many alternations and not enough long runs. Consequently, people's perceptions of chance processes are prone to systematic errors. First, many people think that the probability of heads is greater after a long sequence of tails than after a long sequence of heads; this is the notorious gamblers' fallacy (2). Second, people tend to reject as non-random sequences that contain the expected number of runs because even the occurrence of four heads in a row -- which is quite likely in a sequence of 20 tosses -- makes the sequence appear nonrepresentative (3).

This phenomenon is illustrated in the responses of 100 basketball fans from Cornell and Stanford who were asked to classify sequences representing hits and misses of a basketball player as: *streak shooting, chance shooting, or alternating shooting.* Chance shooting was defined as runs of hits and misses that are just like those generated by coin tossing. Streak shooting and alternating shooting were defined as runs of hits and misses that are longer or shorter, respectively than those observed in coin tossing. All six sequences contained eleven hits and ten misses. They differed in the probability of alternation, (from .4 to .9), that is, the probability that the outcome of a shot would be different from the outcome of the previous shot. In a random sequence the probability of alternation is .5; alternate shooting and streak shooting arise, respectively, when the probability of alternation is less than or greater than .5.

The percentage of "streak" and "chance" responses is presented in Figure 1. As expected, the tendency to perceive a sequence as streak shooting decreases with the probability of alternation. The perception of chance shooting, however, is strongly biased against long runs. The sequences selected as best examples of chance shooting had probabilities of alternation of .7 and .8 instead of .5. Furthermore, the proper example of chance shooting, with a .5 probability of alternation, was classified as streak shooting by 62 subjects and as chance shooting by only 32.

If people misclassify random shooting as streak shooting, they are likely to perceive a positive correlation between the outcomes of successive shots, even when these outcomes are uncorrelated. Indeed, players, reporters and spectators of professional basketball often refer to a player as "streak shooter" or say that he has the "hot hand". These expressions reflect a belief that the player in question is shooting significantly better than expected by chance, given his overall record. This study investigates the nature and the validity of these beliefs.

Prior to the classification of sequences we asked our 100 respondents, all of whom play and watch basketball regularly, several questions about professional basketball. The responses revealed considerable agreement: 91 believed that a player has "a better chance of making a shot after having just made his last two or three shots than he does after having just missed his last two or three shots"; 68 expressed the same belief regarding free throws; 96 thought that "after having made a series of shots in a row ... players tend to take more shots than they normally would"; 84 believed that "it is important to pass the ball to someone who has just made several shots in a row". When asked to consider a hypothetical player who shoots 50% from the field, the average estimate of his field goal percentage was 61% "after having just made a shot", and 42% "after having just missed a shot".

Professional basketball players share these beliefs. Following a team practice session we interviewed seven players and the coach of the Philadelphia 76ers. Six of the players reported that they have on occasion felt that after having made a few shots in a row they "know" that they are going to make their next shot – that they "almost can't miss". Most players also stated that one "has a better chance of making a shot after having just made his last two or three shots than he does after having just missed his last two or three shots". When asked about their own shooting statistics, the mean estimate of field goal percentage for shots taken after a hit (62.5%) was higher than the mean estimate for shots taken after a miss (49.5%). All players agreed that "after having made a series of shots in a row" they "tend to take more shots than they normally would" and that it is important "for the players on a team to pass the ball to someone who has just made several shots in a row". Thus, basketball players and fans appear to believe in the "hot hand" or in "streak shooting".

To test whether basketball players actually shoot in streaks, field goal records of individual players were obtained for 48 home games of the Philadelphia 76ers and their opponents during the 1980-81 season. Table 1 presents, for the nine major players of the Philadelphia 76ers, the probability of a hit conditioned on 1, 2 and 3 hits (misses). The overall shooting percentage for each player, and the number of shots he took are presented in column 5. A comparison of columns 4 and 6 indicate that for eight of the nine players the probability of a hit is actually higher following a miss (mean .54) than following a hit (mean .51), contrary to the streak shooting hypothesis. Column 9 presents the (serial) correlations between the outcomes of successive shots. These correlations were not significantly different than zero for all but one player (Dawkins) whose correlation was negative. Comparisons of the other matching columns (7 vs. 3, and 8 vs. 2) provide further evidence against streak shooting. Additional analyses showed that the probability of a hit (mean .57) following a "cold" period (0 or 1 hits in the last 4 shots) was higher than the probability of a hit (mean .50) following a "hot" period (3 or 4 hits in the last 4 shots). Finally, the observed number of runs in the players' shooting records did not depart from chance expectation except for one player (Dawkins) whose data, again, run counter to the streak shooting hypothesis. Similar conclusions were obtained in the analysis of data from two other NBA teams, the New Jersey Nets and the New York Knicks (4).

Note that "streak shooting" entials a positive serial correlation even if "hot" and "cold" stretches are embedded in larger stretches of normal performance. The observed negative correlations may be explained by shot selection or by defensive strategies. Following a hit a player may become more confident and attempt more difficult shots than after a miss. Similarly, a player who has made several shots in a row is likely to be guarded more closely (because he is believed to be hot) making it harder for him to score again. We next examine other data that are free from the effects of shot selection and defensive strategy. These factors, however, cannot explain or justify the erroneous belief of players and fans that the outcomes of successive shots are positively correlated.

Free throws are shot -- commonly in pairs -- from the same location and without defensive pressure. According to the hot hand hypothesis, players should hit a higher percentage of their second free throws after having made their first free throw than after having missed their first free throw. We obtained data for all pairs of free throws by Boston Celtics players during the 1980-81 and the 1981-82 seasons. The data revealed no evidence that the outcome of the second shot depends on the outcome of the first shot. The correlation was negative for five players and positive for the remaining four and in no case did it approach statistical significance (mean $r = .01$).

As an alternative method for eliminating the effects of shot selection and defensive pressure we recruited 14 members of the men's varsity team and 12 members of the women's varsity team at Cornell to participate in a controlled shooting study. For each player, we determined a distance from which his or her shooting percentage is roughly 50% and we drew two 15 foot arcs at this distance from which each player took 100 shots, 50 from each arc. When shooting baskets, the players were required to move along the arc so that consecutive shots were never taken from exactly the same spot. The centers of the arcs and the basket formed an equilateral triangle.

The analysis of the Cornell data parallels that of the 76ers. The overall probability of a hit following a hit was .47 and the probability of a hit following a miss was .48. The serial correlation was positive for 12 players and negative for 14 (mean r = .015). With the exception of one player who produced a significant positive correlation (.37), both the serial correlations and the distribution of runs provided strong evidence for the hypothesis that the outcomes of successive shots are statistically independent.

We also asked the Cornell players to predict their hits and misses by betting on the outcome of each upcoming shot. Before every shot, each player chose whether to bet *high* in which case he or she would win 5c for a hit and lose 4c for a miss, or to bet *low* in which case he or she would win 2c for a hit and lose 1c for a miss. The players were advised to bet high when they felt confident in their shooting ability and to bet low when they did not. We also obtained betting data from another player who observed the shooter, and decided, independently, whether to bet high or low on each trial. The players' payoff included the amount of money won or lost on the bets made as shooters and as observers.

The players were generally unsuccessful in predicting their performance. The average correlation between the shooters' bets and their performance was .02 and the highest positive correlation was .22. The observers were also unsuccessful in predicting the shooter's performance (mean r = .04). The bets made by both shooters and observers, however, were correlated with the the outcome of the shooters' previous shot (mean r = .40 for the shooter and .42 for the observer). Evidently, both shooters and observers relied on the outcome of the previous shot, in accord with the hot hand hypothesis. Because the correlation between successive shots was negligible (mean r = .01), this betting strategy was not superior to chance, thought it produced a moderate agreement (mean r = .22) between the bets of the shooters and the observers.

The present study shows that, contrary to common belief, the outcome of a shot is largely independent of the preceding one. The discrepancy between the observed basketball statistics and the intuitions of highly interested and informed observers can be traced to a general misconception that small as well as large sequences are representative of their generating process (1). This belief induces the expectation that random sequences will be far more balanced than expected by chance, and the misperception of random sequences as consisting of clusters or streaks (3). This hypothesis explains both the formation and the maintenance of the observed bias. If independent sequences are perceived as streak shooting, no amount of exposure to such sequences will convince the player, the coach, or the fan that the sequences are actually independent. In fact, the more basketball one watches, the more one encounters "evidence" of streak shooting. The misconception of chance has direct consequences for the conduct of the game. Passing the ball to the hot player, who is guarded closely by the opposing team, may be a non-optimal strategy if other players who do not appear hot have a better chance of scoring. Like other cognitive illusions (5), the belief in the hot hand could be costly.

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References and Notes

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Table 1. Probability of making a shot conditioned on the outcome of previous shots for nine members of the Philadelphia 76ers.

Player	P (Hit/3 Misses)	P(Hit/2 Misses)	P(Hit/1 Miss)	P(Hit)	P(Hit/1 Hit)	P(Hit/2 Hits)	P(Hit/3 Hits)	Serial Correlation r
Clint Richardson	.50	.47	.56	.50 (248)	.49	.50	.48	-.020
Julius Erving	.52	.51	.51	.52 (884)	.53	.52	.48	.016
Lionel Hollins	.50	.49	.46	.46 (419)	.46	.46	.32	-.004
Maurice Cheeks	.77	.60	.60	.56 (339)	.55	.54	.59	-.038
Caldwell Jones	.50	.48	.47	.47 (272)	.45	.43	.27	-.016
Andrew Toney	.52	.53	.51	.46 (451)	.43	.40	.34	-.083
Bobby Jones	.61	.58	.58	.54 (433)	.53	.47	.53	-.049
Steve Mix	.70	.56	.52	.52 (351)	.51	.48	.36	-.015
Daryl Dawkins	.88	.73	.71	.62 (403)	.57	.58	.51	** -.142
Weighted Mean =	.56	.53	.54	.52	.51	.50	.46	-.039

The number of shots taken by each player is given in parentheses in Column 5.

* p. .05
**p .01

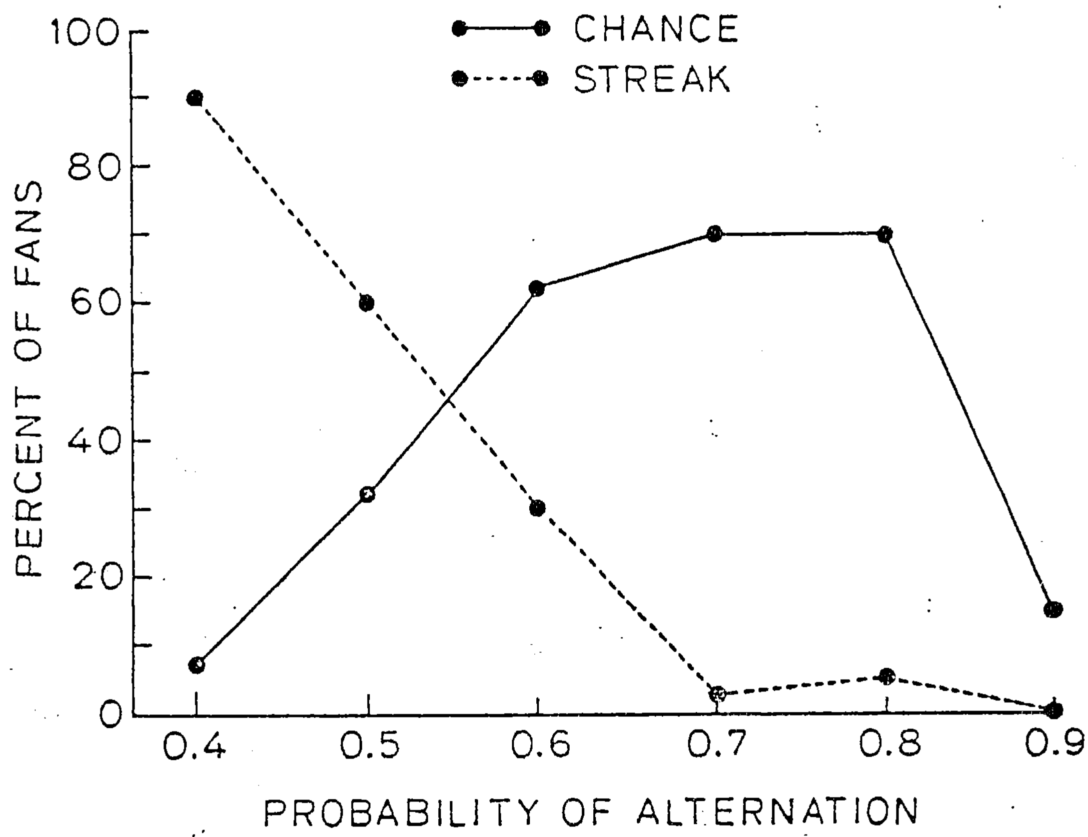

Figure 1. Percentage of basketball fans classifying sequences of hits and misses as examples of Streak Shooting or Chance Shooting, as a function of the probability of alternation within the sequences.