

## Abstract

The combining of information: Investigating and synthesizing what is possibly common in clinical observations or studies via likelihood.

A thesis submitted by Keith O'Rourke of Worcester College towards a D.Phil. degree in the Department of Statistics, University of Oxford, Trinity Term, 2003.

### 0.0.1 Importance sampling approximations for observed summary likelihoods

The lack of closed form formulas for observed summary likelihoods also presented a challenge for this thesis. Initially motivated by a formula given by Barndorff-Nielsen for the analytical derivation of marginal likelihoods, it was realized that rescaled importance sampling allowed the calculation of a likelihood surface when conditional samples were drawn from an "opportunistically" chosen single point in the parameter space. The result used was from Barndorff-Nielsen[1] and simply given (in different notation) as

$$\frac{f_U(u | \theta)}{f_U(u | \theta_0)} = \int \frac{f_X(x | \theta)}{f_X(x | \theta_0)} f_{X|U}(x | u, \theta_0) dx$$

Now, the marginal distribution is simply

$$f_U(u | \theta) = \int_{x^*} f_X(x | \theta) dx \text{ where } x^* \text{ is the level set given by } u = U(\mathbf{x})$$

(or more formally  $x \in \{\mathbf{x} : U(\mathbf{x}) = u\}$ ) but only the (relative) likelihood  $\frac{f_U(u|\theta)}{f_U(u|\theta_0)}$  is needed. Now

$$\frac{f_U(u | \theta)}{f_U(u | \theta_0)} = \int_{x^*} f_X(x | \theta) dx \frac{1}{f_U(u | \theta_0)}$$

$$\frac{f_U(u | \theta)}{f_U(u | \theta_0)} = \int_{x^*} \frac{f_X(x | \theta)}{f_X(x | \theta_0)} \frac{f_X(x | \theta_0)}{f_U(u | \theta_0)} dx$$

$$\frac{f_U(u | \theta)}{f_U(u | \theta_0)} = \int \frac{f_X(x | \theta)}{f_X(x | \theta_0)} \frac{f_X(x | \theta_0)}{f_U(u | \theta_0)} f_{U|X}(u | x, \theta_0) dx$$

$$\frac{f_U(u | \theta)}{f_U(u | \theta_0)} = \int \frac{f_X(x | \theta)}{f_X(x | \theta_0)} f_{X|U}(x | u, \theta_0) dx$$

Alternatively, starting out as importance sampling

$$f_U(u | \theta) = \int \frac{f_X(x | \theta)}{f_{X|u}(x | u, \theta_0)} f_{X|u}(x | u, \theta_0) dx$$

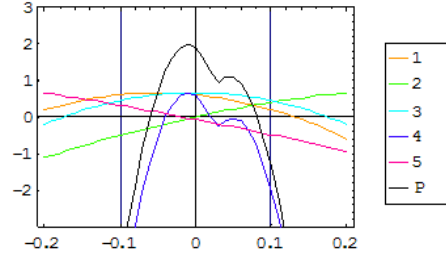


Figure 1: Common Mean: Individual and Pooled Profile Log-Likelihoods, common SDs

$$\frac{f_U(u | \theta)}{f_U(u | \theta_0)} = \int \frac{f_X(x | \theta)}{f_X(x | \theta_0)} f_{X|u}(x | u, \theta_0) dx$$

Conditional samples were simply generated by rejection sampling. An assumed probability distribution for the unobserved sample values was set to opportunistically chosen parameter values and a sample of the same size drawn and only those matching the reported summaries within a given tolerance were kept. An overly high rejection rate here may be suggestive of the assumed probability distribution for the unobserved sample values being inappropriate. Likelihoods for each of the kept samples were then added (no need to normalize) to get an approximation of the observed summary likelihood.

An example in Figure 1 where studies sometimes just reported group means and ranges.

## References

- [1] BARNDORFF-NIELSEN, O. *Information and exponential families: In statistical theory*. Chichester, New York, 1978.